Assessing the Probability of Bankruptcy

Stephen A. Hillegeist* Elizabeth K. Keating†
Donald P. Cram‡ Kyle G. Lundstedt§

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*Kellogg School of Management, Northwestern University. Corresponding author: Stephen Hillegeist, 2001 Sheridan Road, Room 6223, Evanston, IL 60208. Ph. (847) 491-2664; E-mail: s-hillegeist@kellogg.northwestern.edu

†Kennedy School of Government, Harvard University

‡California State University, Fullerton

§VaRisk, Inc.

Abstract

This paper assesses whether two popular accounting-based measures, Altman's (1968) *Z-Score* and Ohlson's (1980) *O-Score*, effectively summarize publicly-available information about the probability of bankruptcy. We compare the relative information content of these *Scores* to a market-based measure of the probability of bankruptcy that we develop based on the Black-Scholes-Merton option-pricing theory, *BSM-Prob*. Our tests show that *BSM-Prob* provides significantly more information than either of the two accounting-based measures. This finding is robust to various modifications of *Z-Score* and *O-Score*, including updating the coefficients, making industry adjustments, and decomposing the *Score* variables into their lagged levels and changes. We recommend that researchers use *BSM-Prob* instead of *Z-Score* and *O-Score* in their studies.

1 Introduction

Academics in the fields of accounting and finance have actively studied bankruptcy prediction since the work of Beaver (1966, 1968) and Altman (1968). With few exceptions, this literature has relied on accounting-based measures as the predictor variables. More recent studies have used proxies for the probability of bankruptcy (PB) as an independent variable rather than as the dependent variable. These latter studies frequently obtain their PB proxies from the existing bankruptcy prediction literature, and hence, have relied on accounting-based measures. Many of these studies have used composite measures that statistically combine several different accounting variables, with Altman's (1968) *Z-Score* and an *O-Score* derived from Ohlson's (1980) Model 1 being the most popular.[1] This paper assesses the performance of these accounting-based composite measures in explaining the cross-sectional variation in the actual probability of bankruptcy.[2] We compare their performance to a market-based PB measure that is derived from the Black-Scholes-Merton option-pricing model.

There are several reasons to question the effectiveness of PB measures that are based on accounting data. While PB estimates are statements about future events, the financial statements are designed to measure past performance, and thus, may not be very informative about the future status of the firm. Financial statements are formulated under the going-concern principle, which assumes that firms will not go bankrupt. Thus, their ability to accurately and reliably assess the probability of bankruptcy will be limited by design. Additionally, the conservatism principle often causes asset values to be understated relative to their market values, particularly for fixed assets and intangibles. Downward-biased asset valuations will cause accounting-based leverage measures to be overstated. These aspects of the accounting system will limit the performance of any accounting-based PB measure.

Another important deficiency of accounting-based bankruptcy prediction models is their failure to incorporate a measure of asset volatility. Volatility is a crucial variable in bankruptcy prediction because it captures the likelihood that the value of the firm's assets will decline to such an extent that the firm will be unable to repay its debts. *Ceteris paribus*, the probability of bankruptcy is increasing with volatility. Two firms with identical leverage ratios can have substantially different PBs depending on their asset volatilities. Therefore, volatility is an important omitted variable in both the Altman (1968) and Ohlson (1980) bankruptcy prediction models.[3]

The stock market provides an alternative and potentially superior source of information regarding PB because it aggregates information from the financial statements as well as from other sources. While the potential for market-based variables to provide information about PB has long been recognized (Beaver (1968)), one difficulty with this approach has been how to extract the PB-related information from market prices. As noted previously in the accounting literature (Cheung (1991), Dhaliwal, Lee, and Fargher (1992)), option-pricing models provide a natural starting point. Based on the approach developed by Black and Scholes (1973) and Merton (1974) (BSM), the firm's equity can be viewed as a call option on the value of the firm's assets. When the value of the assets is below the face value of liabilities (i.e., the strike price), the call option is left unexercised, and the bankrupt firm is turned over to its debtholders. As discussed in Section 2, the probability of bankruptcy is embedded the BSM option-pricing model. We refer to our empirical estimate of this probability as the "Black-Scholes-Merton Probability of Bankruptcy" (BSM-$Prob$). The primary variables used to estimate BSM-$Prob$ are the market value of equity, the standard deviation of equity returns, and total liabilities.

The main advantages of using option-pricing models in bankruptcy prediction are that they provide guidance about the theoretical determinants of bankruptcy risk and they supply the necessary structure to extract bankruptcy-related information from market prices. These potential benefits come at the cost of relying on the models' simplifying assumptions, many of which do not hold in practice. These assumptions can introduce errors and biases into the resulting PB estimates. We discuss how violations of these assumptions are likely to impact the performance of BSM-$Prob$ in Section 5. Another potential shortcoming of the option-based approach is that the stock market may not efficiently impound all publicly-available information about PB into prices. In particular, prior studies suggest that the market does not accurately reflect all of the information in the financial statements. Thus, whether a market-based PB measure derived from an option-pricing model or an accounting-based PB measure performs better is ultimately an empirical question.

In this paper, we empirically compare the performance of *BSM-Prob* to four accounting-based PB measures: *Z-Score* and *O-Score* using the original coefficients and $Z\text{-}Score^u$ and $O\text{-}Score^u$ using updated, but out-of-sample, coefficient estimates. Consistent with much of the prior literature, we examine each measure's ability to explain bankruptcy outcomes over the following year. Our sample consists of 78,100 firm-year observations and 756 initial bankruptcies during the 1980-2000 period. Unlike prior bankruptcy studies that rely on prediction-oriented tests to assess performance, we employ relative information content tests to compare the out-of-sample performance of the five PB measures. As discussed in Section 4.2, these tests use the log likelihood statistics associated with each PB measure to compare their information content. An important advantage of this approach is that it allows us to determine whether the differences in performance are statistically significant.

We estimate discrete hazard models to assess how well each PB measure fits the data and use Vuong (1989) tests to statistically compare the log likelihood statistics of the non-nested models. Our results from this analysis show that *BSM-Prob* contains significantly more information about the probability of bankruptcy (at the 1% level) than any of the accounting-based measures. A comparison of each model's pseudo-R^2 shows that *BSM-Prob* outperforms the original *Z-Score* and *O-Score* by 71% and 33%, respectively. *BSM-Prob*'s pseudo-R^2 is also 20% larger than the psuedo-R^2 for the best accounting-based model (the updated $O\text{-}Score^u$). Our findings are robust to various decompositions of the accounting-based measures that are designed to improve their explanatory power. Overall, our results imply that studies using the *Z-Score* and/or *O-Score* may lack sufficient statistical power to yield reliable results. We provide a copy of the SAS program that calculates *BSM-Prob* in Appendix A.

As discussed above, asset volatility is an important determinant of the likelihood of bankruptcy and it is a key component of *BSM-Prob*. Furthermore, the absence of any volatility measure in the accounting-based models likely leads to a substantial reduction in their performance as firms exhibit considerable cross-sectional variation in volatility (Campbell et al. (2001)). In addition to asset volatility, the other primary component of *BSM-Prob* is a market-based leverage ratio. Although the Altman (1968) model includes a similar market-based leverage variable, both *Z-Score* and *Z-Score*^{u} perform quite poorly. While not conclusive, we believe that these observatitions indicate that the asset volatility component of *BSM-Prob* is primarily responsible for its superior performance.

In addition to significantly outperforming the accounting-based measures, the theory-based *BSM-Prob* affords researchers significant flexibility in their research designs. This flexibility occurs because *BSM-Prob* is computed independently for any publicly-traded firm using a theoretically-derived equation. In contrast, accounting-based PB measures are computed using coefficients that are estimated in a first-stage regression. These coefficients are specific to the characteristics of the sample used in the regression (i.e., time horizon of the estimates, industry, accounting rules, etc.) and their generalizability to other samples is limited.[4]

The paper continues as follows: Section 2 develops *BSM-Prob*. Our sample selection and variable estimation procedures along with the descriptive statistics are reported in Section 3. Section 4 describes our research methodology; and the analyses and results are presented in Section 5. Section 6 summarizes and concludes the paper.

2 BSM and the Probability of Bankruptcy

An important observation in Merton (1974) is that equity can be viewed as a call option on the value of the firm's assets. Equity holders are the residual claimants to the firm's assets and are only subject to limited liability when the firm is bankrupt. Thus, the payoffs to equity mimic the payoffs for call options. Under the *BSM* framework, the strike price of the call option is equal to the face value of the firm's liabilities and the option expires at time T when the debt matures.[5] At time T, equity holders will exercise their option and pay off the debtholders if the value of the firm's assets is greater than the face value of its liabilities. Otherwise, the equity holders will let their call option expire when the value of the assets is not sufficient to fully repay the firm's debts. In this case, the firm files for bankruptcy; ownership is assumed to be transferred costlessly to the debtholders, and the payoff for equity holders is zero. The probability of each outcome, of course, is an important determinant of the value of the call option, and these probabilities are embedded in the BSM model.[6]

The BSM equation for valuing equity as a European call option on the value of the firm's assets is given in eq. 1 below. This equation is modified for dividends and reflects that the stream of dividends paid by the firm accrue to the equity holders.

$$V_E = V_A e^{-\delta T} N(d_1) - X e^{-rT} N(d_2) + (1 - e^{-\delta T}) V_A \tag{1}$$

where $N(d_1)$ and $N(d_2)$ are the standard cumulative normal of d_1 and d_2, respectively, and

$$d_1 = \frac{\ln\left[\frac{V_A}{X}\right] + \left(r - \delta + \frac{\sigma_A^2}{2}\right) T}{\sigma_A \sqrt{T}} \tag{2}$$

and

$$d_2 = d_1 - \sigma_A \sqrt{t} = \frac{\ln\left[\frac{V_A}{X}\right] + \left(r - \delta - \frac{\sigma_A^2}{2}\right) T}{\sigma_A \sqrt{T}} \tag{3}$$

V_E is the current market value of equity; V_A is the current market value of assets; X is the face value of debt maturing at time T; r is the continuously-compounded risk-free rate; δ is the continuous dividend rate expressed in terms of V_A, and σ_A is the standard deviation of asset returns.

The dividend rate, δ, appears twice in the RHS of eq. 1. The $V_A e^{-\delta T}$ term accounts for the fact that the value of the assets at time T will be reduced by the value of the dividends that are distributed up until time T. The addition of the $(1 - e^{-\delta T})V_A$ term is necessary because it is the equity holders that receive the dividends. This term equals zero when

$\delta = 0$. This term does not appear in the traditional equation for valuing a call option on a dividend-paying stock because dividends do not accrue to option holders.

Under the BSM model, the probability of bankruptcy is simply the probability that the market value of assets, V_A, is less than the face value of the liabilities, X, at time T (i.e., $V_A(T) < X$). The BSM model assumes that the natural log of future asset values is distributed normally as follows, where μ is the continuously-compounded expected return on assets:

$$\ln V_A(\mathrm{t}) \sim N[\ln V_A + (\mu - \delta - \frac{\sigma_A^2}{2})t, \sigma_A^2 t] \tag{4}$$

As shown in McDonald (2002, p. 604), the probability that $V_A(T) < X$ is as follows:

$$N\left(-\frac{\ln \frac{V_A}{X} + (\mu - \delta - \frac{\sigma_A^2}{2})T}{\sigma_A \sqrt{T}}\right) = BSM\text{-}Prob \tag{5}$$

Eq. 5 shows that the probability of bankruptcy is a function of the distance between the current value of the firm's assets and the face value of its liabilities $\left(\frac{V_A}{X}\right)$ adjusted for the expected growth in asset values $(\mu - \delta - \frac{\sigma_A^2}{2})$ relative to asset volatility (σ_A). Note that the value of the call option in eq. 1 is not a function of μ. Eq. 1 is derived under the assumption of risk-neutrality where all assets are expected to grow at the risk-free rate, and hence, only the risk-free rate enters into the call option equation. However, the probability of bankruptcy depends upon the actual distribution of future asset values, which is a function of μ.

To empirically estimate $BSM\text{-}Prob$ from eq. 5, we must estimate the market value of assets, V_A, asset volatility, σ_A, and the expected return on assets, μ, since these values are not directly observable. As described below, we first simultaneously estimate V_A and σ_A, and then use these values to estimate μ. Once these steps are completed, we use eq. 5 to calculate the probability of bankruptcy according to the BSM model.

In the first step, we estimate the values of V_A and σ_A by simultaneously solving the call option equation (eq. 1) and the optimal hedge equation $\left[\sigma_E = \frac{V_A e^{-\delta T} N(d_1)\sigma_A}{V_E}\right]$. V_E is set equal to the total market value of equity based on the closing price at the end of the firm's fiscal year. σ_E is computed using daily return data from CRSP over the entire fiscal year. The strike price X is set equal to the book value of total liabilities, T equal to one year, and r is the one-year Treasury bill rate. The dividend rate, δ, is the sum of the prior year's common and preferred dividends divided by the approximate market value of assets, which is defined as total liabilities plus the market value of equity.[7] We use the SAS program in Appendix A to solve the two equations simultaneously for the two unknown variables, V_A and σ_A. The starting values are determined by setting V_A equal to book value of liabilities plus the market value of equity and $\sigma_A = \frac{\sigma_E V_E}{V_E + X}$. The iterative process uses a Newton search algorithm that ends when the pair of values solves both equations. In almost all cases, the process converges within five iterations.

In the second step, we estimate the expected market return on assets, μ, based on the actual return on assets during the previous year. This process is based on the estimates of V_A that were computed in the previous step. In many cases, the actual return on assets is negative. Since expected returns cannot be negative, we set the expected growth rate equal to the risk-free rate in these cases.[8] Thus, $\mu(t)$ is calculated as follows:

$$\mu(t) = \max\left[\frac{V_A(t) + dividends - V_A(t-1)}{V_A(t-1)},\ r\right] \tag{6}$$

where *dividends* is the sum of the common and preferred dividends declared during the year.

Finally, we use the values for V_A, σ_A, μ, δ, T, and X to calculate *BSM-Prob* for each firm-year via eq. 5. To do this, we first calculate the value inside the parentheses in eq. 5. We then determine the probability of bankruptcy corresponding to this value using the standard normal distribution. Note that this procedure ensures that *BSM-Prob* is calculated on an *ex ante* basis so that *BSM-Prob* is always an out-of-sample estimate.

Several commercial vendors provide default probabilities based on option-pricing models, with KMV, LLC being the most well-known. Vassalou and Xing (2003) have also used a similar approach to examine whether the probability of default is a priced risk factor. While the basic approaches are all similar, the three implementations differ in several important aspects. KMV bases its proprietary Expected Default Frequency (EDF) measure on an extension of the BSM model that, among other things, allows for a more complicated capital structure. Along with Vassalou and Xing (2003), KMV arbitrarily defines the strike price X as the sum of short-term liabilities and one-half of long-term liabilities. Instead of using the normal distribution, $EDFs$ are calculated using an empirical distribution of actual defaults based on KMV's large, proprietary database. KMV also uses proprietary methods to estimate V_A, σ_A, and μ. While Vassalou and Xing (2003) rely on the BSM model as we do, they do not adjust for dividends, and their method for estimating μ will frequently result in negative expected growth rates, a result that is inconsistent with asset pricing theory. In addition, they estimate the values for V_A and σ_A using an iterative process that is much more computationally intensive than our method.

3 Data and Variable Estimation

3.1 Sample Selection

Bankruptcy filings between 1980 and 2000 were identified from the Moody's Default Risk Services' Corporate Default database and SDC Platinum's Corporate Restructurings database. We choose 1980 as the earliest year for identifying bankruptcy filings since the Bankruptcy Reform Act of 1978 likely caused the associations between accounting variables and PB to shift. Among other things, the 1978 Act allowed for greater strategic use of bankruptcy, significantly broadened what constitutes a valid claim, combined Chapters X, XI and XII into a single Chapter 11, and substantially shifted power from the bankruptcy judge to the Creditors' Committee (Delaney (1992)). We do not include bankruptcies in 1979 in our sample since our tests are based on out-of-sample performance.

Consistent with the prior literature, we examine the probability of a firm's initial filing for bankruptcy and eliminate any observations for a firm after it has filed for bankruptcy during our sample period. We also limit our sample to industrial firms following Altman (1968) and Ohlson (1980). Our final bankruptcy sample consists of 756 initial bankruptcies and is larger and more complete compared to previous studies. Early studies relied on small samples due, in part, to the lower rate of bankruptcy filings that prevailed before the Bankruptcy Reform Act of 1978 (Delaney (1992)). For example, Altman's (1968) sample contains 33 bankruptcies, and Ohlson (1980) has 105 bankruptcies. More recent studies have employed moderate-sized samples. For example, Shumway (2001) relies on 300 performance-related delistings over the 1962-1992 period.

Non-bankrupt firms consist of all industrial firms that have the CRSP and Compustat data necessary to calculate *BSM-Prob*, *Z-Score*, and *O-Score*. Our final combined sample of solvent and bankrupt firms consists of 78,100 firm-year observations representing 14,303 individual firms. Our larger sample should better reflect the actual composition of firms that were publicly traded during the sample period, thereby improving the accuracy of the coefficient estimates and increasing the power of our statistical tests relative to prior studies.

Table 1 provides summary statistics for our sample. Panel A shows that the average annual bankruptcy rate during our sample period was 0.97%. As expected, the annual rates show considerable fluctuation with the two highest rates in 2000 and 1999 (2.25% and 1.89%, respectively). The 2000 rate is almost five times as large as the lowest rate of 0.48% in 1993. These statistics show that even in the worst years, bankruptcy is a fairly rare event among publicly traded firms. The data indicate that the annual failure rates generally reflect the overall health of the economy, with relatively high rates during the recessions of the early 1980s and late 1990s and lower rates during the expansion years of the early and mid-1990s.

The bankruptcy rate also varies considerably by industry. Based on the Fama and French (1997) industry classification system, Panel B provides industry failure rates based on both the number of individual firms and the number of firm-years. Panel B shows that the coal (19.23%), textile (15.83%), and retail (11.48%) industries have experienced the highest rates of failure, measured as the percentage of firms in the industry that filed for bankruptcy during our sample period. At the opposite end of the spectrum, no publicly-traded firms in the agricultural, beer, guns, non-beer beverages, and tobacco industries filed for bankruptcy between 1980 and 2000.

3.2 Estimation of *Z-Score* and *O-Score*

Our analysis of accounting-based PB measures is based on the *Z-Score* and the *O-Score*. We focus on these two measures due to their widespread use in the prior literature (see footnote 1 for a partial list of studies) as well as for their ability to summarize a number of different accounting-based variables. Altman (1968) considered various combinations of 22 variables before choosing the five with the highest predictive power. The Ohlson model consists of nine accounting-based variables. The *Z-* and *O-Scores* are calculated using fiscal year-end data and are used to measure the risk of bankruptcy over the twelve-month period beginning four months after a firm's fiscal year end. The *Z-* and *O-Scores* are computed as the fitted values using the original coefficients. Computed this way, the accounting-based *Scores* do not represent bankruptcy probabilities, but can be turned into probabilities using the logistic transformation.

The Altman (1968) coefficients were estimated using data between 1946 and 1965, and Ohlson (1980) relied on data between 1970 and 1976. There are a number of reasons why the associations between the accounting variables and the actual probability of bankruptcy may have changed since the original sample periods. As discussed earlier, the Bankruptcy Reform Act of 1978 represents a significant change in the legal environment and was followed by an increase in the number of strategic bankruptcy filings. Another factor is the large number of asbestos-related lawsuits that have resulted in 67 bankruptcies through 2000 (Warren (2003)). Significant intervening changes in accounting rules will also have changed the original associations through differences in how the accounting variables are measured.

To allow for changes in the estimated coefficients over time, we also evaluate the performance of the Altman (1968) and Ohlson (1980) models using updated coefficients that are estimated using an expanding rolling window approach. For the first window, we estimate updated coefficients using firm-year observations from 1979 and bankruptcy outcomes in 1980. These updated coefficients are combined with accounting data from 1980 to explain bankruptcy outcomes in 1981. For the second window, we use firm-year observations from 1979 and 1980 and bankruptcy outcomes from 1980 and 1981 to estimate a second set of updated coefficients. This second set of coefficients is combined with accounting data from 1981 to explain bankruptcy outcomes in 1982. This process continues on so that the updated coefficients used to explain bankruptcy outcomes in 2000 are based on firm-year observations from 1979 to 1998 and bankruptcies from 1980 to 1999. This approach ensures that all of the updated coefficients are estimated out-of-sample relative to the evaluation periods. Expanding the size of the window each year allows us to incorporate all available data, which should increase the efficiency and accuracy of our coefficient estimates. This procedure differs from the sample selection procedures used in the original studies where only one firm-year observation was used for each firm.

The expanding rolling window approach results in 20 sets of updated coefficients for each accounting-based model. We refer to the PB measures using the updated coefficients as $Z\text{-}Score^{\text{u}}$ and $O\text{-}Score^{\text{u}}$, respectively, and reserve $Z\text{-}Score$ and $O\text{-}Score$ for the original coefficients. The updated coefficients are presented in Table 2. For expositional reasons, we only report one set of coefficients for each model that are estimated using the entire sample. We report the actual coefficients from the original studies for comparability purposes. In Altman (1968), PB is decreasing with the $Z\text{-}Score$, while PB is increasing with both the $O\text{-}Score$ and $BSM\text{-}Prob$. For expositional reasons, we change the signs of the original Altman parameter estimates so that the $Z\text{-}Score$ is increasing with PB.[9]

Consistent with Begley, Ming, and Watts (1996), we find that several of the coefficients in each model have substantially changed from their original values. This finding demonstrates that the associations between PB and accounting variables are not stable over time and suggests caution in their use as PB measures. Surprisingly, we find that only two of the Altman variables are statistically significant, V_E/TL and $EBIT/TA$, while the remaining three accounting-based variables, WC/TA, RE/TA, and S/TA, are insignificant. For the Ohlson model, we find that eight of the nine updated coefficients are statistically significant compared to seven in the original estimation. Furthermore, five of the eight significant coefficients have different signs than their original counterparts. The fact that the changed signs are not intuitive also suggests caution in relying on accounting-based models.

3.3 Descriptive Statistics

Table 3 reports each bankruptcy measure in its probability form to facilitate the comparisons among the five PB measures. For *BSM-Prob*, this is just the value from eq. 5. For the accounting-based *Scores*, we use the logistic cumulative distribution function in eq. 7 (Section 4 below) to convert *Scores* into probabilities, where $probability = \left(\frac{e^{Score}}{1+e^{Score}}\right)$. While this transformation is not strictly correct for the original *Z-Score*, which was estimated using multiple discriminant analysis (MDA), McFadden (1976) shows that the MDA and logit approaches are closely related under normality assumptions. We refer to the probability implied by the original *Z-Score* and *O-Score* as *Z-Prob* and *O-Prob*. We use $Z\text{-}Prob^{\mathrm{u}}$ and $O\text{-}Prob^{\mathrm{u}}$ to indicate that the probability is based on the updated coefficients.

Table 3 shows that the differences in the sample means and medians between the solvent and bankrupt firm-years are all in the expected direction and are all statistically significant at the 1% level. The trailing twelve-month economy-wide bankruptcy rate, *AnnualRate*, is significantly higher for the firms that declare bankruptcy over the following year compared to firms that remain solvent. This significant difference for *AnnualRate* supports its use as the baseline hazard proxy (see Section 4).

While the median estimate for *BSM-Prob* is essentially zero (0.01%) for the solvent firm-years, the mean estimate is substantially higher at 5.61%. This estimate is much higher than the average annual sample bankruptcy rate of 0.97% (Table 1). This indicates that *BSM-Prob* may not be well-calibrated, at least for high PB firms, since it may overestimate the actual probability of bankruptcy. This could be due to three, non-exclusive reasons. First, our estimates implicitly assume that all of the firm's liabilities mature in one year. For most firms, this substantially underestimates the actual duration of the liabilities and can lead to higher *BSM-Prob* estimates. Second, many firms that might otherwise declare bankruptcy are either acquired or liquidated outside of the bankruptcy process. While such firms may meet an economic definition of bankruptcy, they are not classified as such in our sample since we rely on the legal definition of bankruptcy. Third, the *BSM* model assumes that asset returns are normally distributed. While analytically tractable, the normal distribution may not correspond to the actual distribution.

The summary statistics for the accounting-based PB estimates in Table 3 indicate that these models are not well-calibrated either. The mean and median *Z-Prob* and *O-Prob* values for solvent firms are all substantially greater than the historical average rate. The original coefficients in Altman (1968) and Ohlson (1980) were estimated using choice-based samples where the sample percentage of bankrupt observations exceeded the underlying population percentage. As Palepu (1986) and Zmijewski (1984) show, this type of sample selection process will lead to bankruptcy probability estimates that are inconsistent and biased upwards unless modified estimation procedures are used. While this bias will tend to inflate the PB estimates for *Z-Prob* and *O-Prob*, it will not effect *Z-Prob*$^{\mathrm{u}}$ and *O-Prob*$^{\mathrm{u}}$ because their coefficients are based on the entire sample population. Reflecting this lack of bias, the estimated values for *Z-Prob*$^{\mathrm{u}}$ and *O-Prob*$^{\mathrm{u}}$ appear to be more reasonable for the solvent firms with mean and median values near the historical bankruptcy rate. However, their mean and median values for the bankrupt firms are unreasonably low (between 1.20% and 2.66%). Although these probability estimates may not be well-calibrated, their ability to explain the variation in the actual probability of bankruptcy can still be high.

Pearson and Spearman correlations for the primary variables are presented in Table 4. All of the correlations are statistically significant at the 1% level. The variable, *Bankrupt*, is an indicator variable that equals one if the firm declares bankruptcy in the four to sixteen months following its fiscal year end, and zero otherwise. The Spearman correlations in Table 4 show that *BSM-Prob* has a correlation of 0.11 with *Bankrupt*, which is equal to the correlation between *O-Prob* and *Bankrupt*. Interestingly, the original accounting-based *Prob* measures have slightly higher correlations with *Bankrupt* than their updated counterparts do (0.09 and 0.09, respectively). This finding is surprising since the *Prob*$^{\mathrm{u}}$ measures are based on much more current data compared to the original *Prob* measures. Overall, the correlations between the *Prob* measures and *Bankrupt* are relatively low and indicate that their performance in our statistical tests may be limited.

As expected, the Pearson correlations in Table 4 show that all of the *Prob* measures are positively correlated with each other. *BSM-Prob* has a higher correlation with *Z-Prob* and *O-Prob* (0.43 and 0.39, respectively) than with *Z-Prob*^{u} and *O-Prob*^{u} (0.10 and 0.21, respectively). The highest correlation among any two PB measures is between *Z-Prob* and *O-Prob* (0.65), and the lowest correlation is between *BSM-Prob* and *Z-Prob*^{u} (0.10). While the market-based and accounting-based PB estimates are positively correlated with each other, the moderate magnitudes of the correlations suggest that each PB measure may be reflecting different information about the probability of bankruptcy. One area in which the accounting-based measures clearly differ from *BSM-Prob* is that they do not include a measure of volatility, which is a key component of *BSM-Prob*.

4 Methodology

4.1 Testing Procedures

We use relative information content tests to compare the amount of bankruptcy-related information contained in each of the five bankruptcy measures. Relative information tests examine whether one variable (or set of variables) provides more total information compared to another variable (or set of variables). Relative tests are appropriate when rankings by total information content are desired, as in this study. The relative information tests are based on how well each PB measure explains the variation in the actual probability of bankruptcy using a discrete hazard model. As explained below, the discrete hazard model is particularly well-suited for bankruptcy data and offers a number of econometric advantages. We use a logit-based version of the Vuong (1989) test to statistically compare the log likelihood statistics of the five non-nested discrete hazard models.[10] An important advantage of using statistical tests to compare model performance is that it allows us to determine whether differences in performance are statistically significant. Such determinations are not possible using prediction-oriented tests. We believe that this is the first study to compare the performance of alternative bankruptcy measures using relative information content tests.

Contrary to our approach, many prior studies in the bankruptcy literature rely on prediction-oriented tests to distinguish between alternative statistical models and/or different groups of explanatory variables. These tests involve determining a cutoff value that is used to classify which firms are expected to remain solvent and which are expected to declare bankruptcy within a particular (typically one-year) time-horizon. Prediction accuracy is assessed by comparing the total Type I and II error rates for each alternative specification, and the model with the lowest total error rate is deemed the best.

We forego using prediction-oriented tests for several reasons. First, while the prediction error testing methodology presumes a dichotomous decision context, we believe that a decision-maker will more typically make a continuous decision choice. For example, loan officers determine which interest rate to charge on a particular loan, and that interest rate will vary (implicitly or explicitly) with the estimated probability of bankruptcy. Second, the prediction error methodology assumes that the costs of each type of classification error are equal. For instance, this method implies that a regulator will decide whether or not to take over a firm without taking into consideration that the cost of allowing a failing firm to continue may be many times greater than the cost of shutting down a solvent firm. Changing the relative Type I and Type II error costs can result in a different ranking of the models. Since error costs will vary with the decision context, it is not clear what can be concluded from tests that assume a fixed ratio of error costs.

More importantly, we believe that examining classification error rates is not an appropriate method to test the accuracy of probability estimates. To see this, consider the approach taken in Begley, Ming, and Watts (1996) for assessing the accuracy of the PB estimates based on the Ohlson (1980) model. A firm is predicted to declare bankruptcy if the estimated PB is greater than 3.8% and is predicted to remain solvent otherwise. Thus, a prediction error is deemed to occur if a firm with an *ex ante* PB estimate of 5% does not go bankrupt, even though the firm was expected to survive with a 95% probability. The 5% PB estimate for a firm that does not subsequently file for bankruptcy could reasonably be considered a successful prediction of solvency, rather than as an unsuccessful prediction of bankruptcy.

Whether or not a single firm declares bankruptcy *ex post* does not provide evidence about whether the *ex ante* 5% estimate was an accurate reflection of the true probability. Relying on *ex post* outcomes to assess the accuracy of an *ex ante* probability estimate of 5% is only feasible when there are many firms that each had an estimated PB of approximately 5%. In such a case, one could compare whether the actual, *ex post*, percentage of firms that declared bankruptcy was significantly different from the *ex ante* 5% estimate. However, given the historically low annual bankruptcy rates (about 1% per year on average) and the small total number of bankruptcies available, this approach is impractical. For these reasons, we rely on statistical information content tests to assess the alternative PB estimates.

4.2 Discrete Hazard Model

We use a discrete hazard model to assess how well each PB measure explains the actual probability of bankruptcy in our sample. Perhaps a more typical choice would be to use a logit (or probit) model that only includes one firm-year observation for each firm (Ohlson (1980)). In such "single-period" models, one firm-year observation for each non-bankrupt firm is randomly selected from the available firm-years. For bankrupt firms, the firm-year immediately prior to the bankruptcy filing is (non-randomly) selected. The ordinary logit model has the following form where p is the actual probability of bankruptcy, α is a constant, $\mathbf{X}$ is a vector of explanatory variables, and $\boldsymbol{\beta}$ is the coefficient vector:

$$p_i = \frac{e^{\alpha + X_i \boldsymbol{\beta}}}{1 + e^{\alpha + X_i \boldsymbol{\beta}}} \tag{7}$$

There are two econometric problems with the single-period logit approach: (1) a sample selection bias that arises from using only one, non-randomly selected observation per bankrupt firm, and (2) a failure to model time-varying changes in the underlying or baseline risk of bankruptcy that induces cross-sectional dependence in the data. Shumway (2001) demonstrates that these problems can result in biased, inefficient, and generally inconsistent coefficient estimates, and Beck, Katz, and Tucker (1998) show that the standard errors will be understated.

Following the suggestion in Beck, Katz, and Tucker (1998) and Shumway (2001), we use a discrete hazard model to overcome these econometric problems. The discrete hazard model is well-suited to analyze data that consists of binary, time-series, and cross-sectional observations, such as bankruptcy data. The discrete hazard model is related to the logit model and has the following form, where $\alpha(t)$ is a time-varying, system-wide variable that captures the baseline hazard rate:

$$p_{i,t} = \frac{e^{\alpha(t)+X_{i,t}\boldsymbol{\beta}}}{1 + e^{\alpha(t)+X_{i,t}\boldsymbol{\beta}}} \tag{8}$$

The discrete hazard estimator differs from ordinary logit in two important aspects: (1) the subscript t reflects the use of multiple firm-year observations for each firm i, and (2) it includes a time-varying baseline hazard rate, $\alpha(t)$. The inclusion of all available firm-year observations in the hazard model eliminates the sample selection bias discussed above. Including multiple observations for the same firm in the regression can result in understated standard errors. To correct for this effect, we report Huber-White standard errors that are robust to both serial correlation and heteroskedasticity (Rogers (1993)). In addition, using all available firm-years will result in more efficient coefficient estimates since all available data will be used in their estimation.[11]

Fluctuations in the baseline hazard rate will cause observations to be correlated cross-sectionally across time. For example, the baseline hazard rate for bankruptcy will be higher during a recession and lower during an economic expansion. This temporal dependence can result in understated standard errors (overstated t-statistics) in a logit model. The discrete hazard model mitigates this problem by incorporating a time-varying baseline hazard rate in the regression model. Ideally, the hazard rate is incorporated by including the system-level variables, such as macro-economic factors, that cause the temporal dependence in the data. We proxy for the baseline hazard with the economy-wide bankruptcy rate over the previous year, *AnnualRate*. *AnnualRate* is the number of corporate bankruptcies divided by the total number of firms in our sample over the previous 12 months, expressed as a percentage. For these reasons, our statistical methodology should yield unbiased and consistent coefficients and standard errors.

For the discrete hazard model to yield reliable results, the independent variables have to be in a form that is consistent with the model's underlying assumptions. Since independent variables in the form of probabilities, such as $BSM\text{-}Prob$, are not consistent with the discrete hazard model (in addition to logit and probit models), we transform $BSM\text{-}Prob$, into a "score" form using the inverse logistic function, so that $BSM\text{-}Score = \ln\left(\frac{BSM\text{-}Prob}{1-BSM\text{-}Prob}\right)$. As $BSM\text{-}Prob$ approaches zero (one), $BSM\text{-}Score$ approaches negative (positive) infinity. Including such extremely large (absolute) values in the regression will cause very large standard errors and result in low likelihood values. To avoid these statistical problems, we winsorize the sample so that the minimum (maximum) value of $BSM\text{-}Prob$ equals 0.00001 (0.99999). This winsorization causes the minimum (maximum) $BSM\text{-}Score$ value to be -11.51292 (+11.51292). To be consistent, we also winsorize the accounting-based $Scores$ using the same cut-off points. The winsorization process affects 5.3%, 2.2%, 1.1%, 0.1%, and 39.0% of the $Z\text{-}Score$, $Z\text{-}Score^{u}$, $O\text{-}Score$, $O\text{-}Score^{u}$, and $BSM\text{-}Score$ estimates, respectively. The qualitative results reported in Tables 5 - 8 below are robust to using several alternative winsorizing cutoff points.

5 Analysis and Results

Our primary analysis compares the relative information content of $BSM\text{-}Score$ and the four accounting-based PB measures: $Z\text{-}Score$, $Z\text{-}Score^{u}$, $O\text{-}Score$, and $O\text{-}Score^{u}$. We estimate five separate discrete hazard models using the economy-wide rate of bankruptcy over the previous twelve months, $AnnualRate$, as the proxy for the baseline hazard rate. The results of these analyses are presented in Table 5.

In all five models, $AnnualRate$ is positive and significant at the 1% level. This finding demonstrates that the baseline hazard rate provides significant incremental information beyond that provided by each of the five PB measures. These results are consistent with a time-varying underlying bankruptcy hazard reflecting the general macroeconomic conditions that cause cross-sectional dependence among the observations. Previous bankruptcy research has not examined this important factor.

Table 5 shows that *O-Score*^{u} provides relatively more information than any of the other accounting-based PB measures. This superiority is indicated by the *O-Score*^{u} model having the largest log likelihood statistic (-3,831) and pseudo-R^2 (0.10). The original *O-Score* is the second best accounting measure with a log likelihood of -3,881 and a pseudo-R^2 of 0.09. An unreported Vuong test shows that the difference in performance between *O-Score*^{u} and *O-Score* is significant at the 1% level. These findings are inconsistent with Begley, Ming, and Watts (1996), who conclude that researchers should use the original Ohlson coefficients rather than their updated coefficients.

In contrast to the Ohlson model results, Table 5 shows that *Z-Score* outperforms *Z-Score*^{u}. This finding is surprising for two reasons. First, we expect that using coefficients estimated using more recent data should provide a better fit compared to using coefficients based on a small data set from decades earlier. Second, prior research (Lo (1986)) suggests that the logit methodology underlying *Z-Score*^{u} is superior to the MDA methodology underlying the original *Z-Score*. Both of the measures based on Altman (1968) perform significantly worse than either of the Ohlson (1980) measures.

Table 5 shows that the market-based measure, *BSM-Score*, outperforms the best accounting-based measure, *O-Score*^{u}. The difference between the two measures' log likelihood statistics is 103 (-3,831 vs. -3,728). An unreported Vuong test shows that this difference is significant at the 1% level. Additionally, the pseudo-R^2 for the *BSM-Score* model (0.12) is 20% larger than for the *O-Score*^{u} model (0.10) and is twice as large as the pseudo-R^2 for the worst-performing model, *Z-Score*^{u} (0.06). These results suggest that researchers can increase the power of their tests by using *BMS-Prob* instead of *Z-Score* or *O-Score* as their proxy for the probability of bankruptcy.

As discussed in the Introduction, we believe that the asset volatility component of *BSM-Prob* is most responsible for its superior performance. There are no variables measuring volatility in either of the accounting-based models. Another factor contributing to *BSM-*

Prob's superior performance is that because it is based on market prices, *BSM-Prob* reflects the market's broader set of information about the probability of bankruptcy. In addition, the non-linear functional form of *BSM-Prob* is derived from an option-pricing model while the accounting-based models are *ad hoc* and empirically driven. The other primary component of *BSM-Prob* is a market-based leverage ratio. A similar leverage variable is included in the Altman (1968) model.[12] Despite including this market-based variable, both *Z-Score* and *Z-Score*^{u} perform quite poorly. This finding suggests that using a market-based leverage variable is not driving *BSM-Prob*'s superior performance.

Despite its superior performance relative to the accounting-based models, the pseudo-R^2 for the *BSM-Score* is relatively low at 0.12, indicating that much of the out-of-sample variation in the probability of bankruptcy is driven by factors outside of the *BSM* model. One possible explanation for this result is that in estimating PB over the following year, we have implicitly assumed that all firm's liabilities mature in one year. This assumption is clearly violated in practice. We assess the importance of this assumption by examining two alternative specifications of *BSM-Score*. In the first specification, the default point X is set equal to current liabilities. In the second specification, X is set equal to current liabilities plus one-half of long-term liabilities, which is the assumption used in Vassalou and Xing (2003). Lowering the default point will mechanically reduce the PB estimates. Untabulated results indicate that the performances of these two alternative specifications are similar but slightly weaker than the results discussed above. None of the specifications allows for variation in the duration of liabilities. To the extent the true PB is correlated with the duration of the firm's liabilities, the performance of *BSM-Score* will be reduced.

The *BSM* model assumes that if the value of the firm's assets is less than its total liabilities at time T, $(V_A(T) < X)$, the firm simultaneously defaults, declares bankruptcy and costlessly turns control over to the bondholders. Accordingly, *BSM-Prob* measures the probability that $V_A(T) < X$. In practice, bankruptcy does not always occur when this economic condition is met. Frictions in the bankruptcy process, such as violations of strict priority and deadweight costs, can lead to forced debt renegotiations that do not entail a formal bankruptcy filing. The firm can also avoid an immediate bankruptcy filing by meeting its current obligations if some of its liabilities are due at a later date. On the other hand, some firms will file for bankruptcy when they are economically solvent, i.e., $V_A(T) > X$. Firms will strategically enter into bankruptcy "early" to break unfavorable contracts and protect themselves from litigation. Additionally, short term liquidity constraints can prevent the firm from meeting its obligations even though its total liabilities are less than the market value of assets. To the extent that these situations occur, the empirical performance of BSM-$Score$ will be reduced because these possibilities are not incorporated into the BSM model.

It may also be that errors in measuring the BSM model's input variables reduce BSM-$Score$'s empirical performance. For example, the expected market return on assets and expected level of volatility should be used in eq. 5 to calculate the probability of bankruptcy. These expected values are measured with error since they are estimated using historical data. Thus, both model mis-specifications and measurement errors will tend to reduce BSM-$Score$'s ability to explain the probability of bankruptcy.

5.1 Additional Relative Information Content Tests

While the results in Table 5 show that BSM-$Score$ significantly outperforms the four accounting-based PB measures, the original models may not extract all of the available PB-related information. Results in Platt and Platt (1991) suggest that additional PB information can extracted by adjusting the PB measures for industry effects. We investigate whether industry-adjusting alters the relative performance of the five PB measures. We use the Fama and French (1997) industry groupings to divide the sample into the forty-four industries that are described in Table 1, Panel B. We decompose each variable into the prior year's industry mean (IM) and its deviation from the industry mean (ID).

Before comparing the relative information content of the decomposed models, we examine whether the decomposition is effective by comparing the performance of each industry-adjusted model to its aggregated counterpart. The results from unreported Vuong tests indicate that industry-adjusting the *O-Score* and *O-Score*^{u} models successfully increases their performance (significant at the 5% level), but is only moderately successful for the *BSM-Score* model (significant at the 10% level). The results for the industry-adjusted *Z-Score* and *Z-Score*^{u} models show no significant improvements and suggest that the decomposition is unlikely to substantially increase their performance.

The results for the industry-adjusted models are presented in Table 6, Panel A. All of the PB variables are positive and are all statistically significant at the 1% level except for $O\text{-}Score^{u}_{IM}$, which is significant at the 5% level, and $O\text{-}Score_{IM}$, which is not significant. Comparing the relative performance of the five PB measures, Panel A shows that the industry-adjustments have not altered the relative information content rankings from Table 5. The *BSM-Score* model is the best overall model. An unreported Vuong test demonstrates that it significantly outperforms the best accounting model, *O-Score*^{u}, at the 1% level. As before, the *O-Score* model outperforms the *Z-Score* model, and the *Z-Score*^{u} model performs worst.

In Panel B of Table 6, we examine whether the combination of the accounting-based IM and ID variables collectively outperforms the aggregate *BSM-Score* model. This analysis allows us to assess whether the decomposition increases each accounting model's performance vis-a-vis the aggregate market-based measure. The z-statistics from the Vuong tests indicate that *BSM-Score* provides significantly more information than each combination of the industry-adjusted accounting-based measures at the 1% level.[13] Thus, our conclusion from Table 5 that the market-based *BSM-Score* outperforms the four accounting-based measures continues to hold even after they are industry-adjusted.

Another potential method to extract additional PB-related information from the PB measures is to decompose the end-of-year t-1 level of the predictor variable into its beginning-of-year t-1 level and year t-1 annual change (Dambolena and Khoury (1980)). This decomposition reduces the sample size to 65,627 firm-year observations. Similar to above, we first examine whether the decomposition is effective by comparing the performance of each lagged level and changes model to its aggregated current levels counterpart, where both models are estimated using the reduced sample. The results from unreported Vuong tests indicate that none of the decomposed models performs significantly better than the corresponding aggregated models. These results suggest this decomposition is unlikely to substantially increase the performance of the bankruptcy measures.

To investigate whether the superiority of BSM-$Score$ continues to hold after this decomposition, Table 7, Panel A reports the results of regression models that include both the beginning-of-year levels (t-1) and changes (Δ) for each PB measure. The results show that both the t-1 and Δ predictor variables are positive and statistically significant at the 1% level in each regression model. Additionally, the magnitudes of the coefficients are similar to those reported in Table 5, with the exception of Z-$Score^{u}$. Comparing the relative performance of the five models, Panel A shows that the decomposition has not altered the relative information content rankings from Tables 5 and 6. This finding is not surprising since the decompositions did not result in any significant increases in performance. The BSM-$Score$ model is the best overall model, and an unreported Vuong test shows that it significantly outperforms the best accounting model, O-$Score^{u}$, at the 1% level.

We next examine whether the decomposed accounting-based variables contain more information than just the end-of-year t-1 level of BSM-$Score$. The z-statistics from the Vuong tests are reported in Panel B of Table 7 and indicate that by itself, BSM-$Score$ provides more information than the combined lagged levels and changes of the four accounting-based models. The differences in performance are significant at the 1% level for the Z-$Score$, Z-$Score^{u}$ and O-$Score$ models and is significant at the 5% level for the O-$Score^{u}$ model. Thus, our finding that the market-based BSM-$Score$ outperforms the four accounting-based $Score$ variables continues to hold even after we disaggregate them into their lagged levels and changes components.

5.2 Incremental Information Content Tests

The results presented in Table 4 show that the correlations between BSM-$Prob$ and the accounting-based PB measures, while positive, are relatively moderate in magnitude. The low correlations suggest that the PB measures may be capturing different information about the probability of bankruptcy. To assess whether the accounting-based measures reflect PB information beyond that contained in *BSM-Prob*, we examine the incremental information content of the four accounting-based *Scores* using regression models that also include *BSM-Score* and *AnnualRate*.

The results of these regressions are presented in Table 8. In each model, *BSM-Score* is positive and significant at the 1% level, and the coefficient magnitudes are similar across the four models. Additionally, all four of the accounting-based *Score* variables are positive and significant at the 1% level. Unreported Vuong tests indicate that each combined model contains significantly more information (at the 1% level) compared to the *BSM-Score* model from Table 5. Thus, each accounting-based *Score* variable provides a significant amount of additional information about the probability of bankruptcy beyond that contained in *BSM-Score*.

Table 8 shows that the combined *O-Score*^{u} and *BSM-Score* model produces the largest log likelihood statistic (-3,592) and pseudo-R^2 (+0.16). Adding *O-Score* to the *BSM-Score* model results in the second best model (-3,592 and +0.15, respectively). An unreported Vuong test shows that the difference in performance is significant at the 1% level. Therefore, *O-Score*^{u} adds significantly more incremental information to *BSM-Score* compared to *O-Score*.

The results in Table 8 indicate that *BSM-Score* fails to reflect all of the PB information contained in the accounting-based variables. There are several possible explanations for why the accounting-based *Scores* are incrementally informative to *BSM-Score*. One possibility is that the incremental information provided by the accounting-based PB measures is correlated with measurement errors in the proxies for the *BSM* input variables. The accounting-based *Scores* may also be correlated with mis-specifications in the *BSM* model that are caused by its restrictive assumptions. Furthermore, many debt covenants are based on accounting numbers. Core and Schrand (1999) use an option-pricing framework to show how equity valuation will be affected by the presence of accounting-based debt covenants. Since these covenant effects are not captured by the BSM model, its effectiveness will be reduced. Another possibility is that the stock markets are not fully efficient in that they fail to incorporate all of the PB information contained in the financial statements. We leave it to future research to determine why the accounting-based measures contain incremental PB information.

6 Summary and Conclusions

This paper uses a comprehensive bankruptcy database and the discrete hazard rate methodology to compare the relative information content of measures of the probability of bankruptcy based on the Altman (1968), Ohlson (1980), and Black-Scholes-Merton models. Our results demonstrate that the market-based BSM-$Prob$ provides significantly more information about the probability of bankruptcy than do either of the popular accounting-based measures. This conclusion is robust to various modifications of the Z- and O-$Scores$, including the use of updated coefficients based on our sample, adjusting for industry effects, and separating the measures into their lagged level and changes components.

Our results suggest that researchers should use BSM-$Prob$ instead of the traditional accounting-based measures as a proxy for the probability of bankruptcy. In addition to its superior empirical performance, the theory-based BSM-$Prob$ provides researchers with a number of advantages that are not available when the traditional accounting-based measures are used. Since it is based on market prices, BSM-$Prob$ can be estimated at any point in time for any publicly-traded firm regardless of the time period and industry. Our results indicate that incorporating industry and firm-specific time effects does little to alter the performance of BSM-$Prob$. Additionally, while this paper follows the general practice of assessing the probability of bankruptcy over a one-year time horizon, researchers may require a PB estimate for a different time horizon. BSM-$Prob$ is easily modified to compute the probability of bankruptcy over any time horizon by changing the time parameter T in eq. 5. In contrast, accounting-based PB measures are based on coefficients estimated in a first-stage regression, and thus, the estimates depend on the characteristics of the first-stage sample. Finally, unlike accounting-based models that are specific to a single set of accounting rules (country), *BSM-Prob* can be consistently estimated across accounting regimes (countries), and thus facilitates intertemporal (international) comparisons. Consequently, *BSM-Prob* provides researchers with a significant amount of flexibility in their research designs that is not available using accounting-based measures.

An important question is whether the additional explanatory power of *BSM-Prob* is substantial enough to result in different conclusions for research studies that incorporate PB proxies as explanatory variables. We are aware of one research question that has been studied using both types of PB measures. Using *Z-Score* and *O-Score*, Dichev (1998) unexpectedly finds a negative association between returns and the probability of bankruptcy. However, using a different implementation of *BSM-Prob*, Vassalou and Xing (2003) find that, consistent with economic theory, stock returns are positively associated with the probability of bankruptcy. These disparate findings suggest that the choice of which PB measure to use is economically important as it can alter a study's conclusions. Since our findings indicate that *BSM-Prob* is a more powerful PB proxy, we recommend that researchers use it in their studies.

Our research opens the opportunity to explore the role of bankruptcy risk using a more powerful tool. Our results in conjunction with those in Vassalou and Xing (2003) suggest that prior findings in the literature that are based on accounting-based PB measures may require re-examination given their relatively low performance in our study. The relation between bankruptcy risk and the pricing of loans and bonds, the likelihood of an adverse audit opinion, and earnings management are promising areas that could be investigated using *BSM-Prob*. Other studies, including Barth, Beaver, and Landsman (1998), Billings (1999), and Dhaliwal and Reynolds (1994), have used bond ratings to proxy for the probability of bankruptcy. Bond ratings are based on both public information and private information conveyed to the rating agencies by firms. It remains an empirical question whether *BSM-Prob* represents a superior PB measure compared to bond ratings.

7 Appendix A: SAS Code to Calculate BSM-Prob

The following SAS code applies a PROC MODEL statement to calculate the probability of bankruptcy, *BSM-Prob*, following the methodology described in Section 2. We have modified the notation used in the text slightly to conform with SAS conventions that prohibit the use of subscripts and Greek letters. For equity volatility (sige), we compute a daily standard deviation of returns using CRSP data. In addition to Compustat variables, we set the risk-free rate (r) to the one-year t-bill rate, and time is one year.
As starting values for market value of assets (va) and asset volatility (siga), we use:

```
va = x + ve ;
siga = sige * ve / ( x + ve ) ;
```

The dividend rate is computed as follows:

```
divrate = ( data19 + data21 ) /( x + ve ) ;
if divrate < 0 then divrate =. ;
if divrate >1 then divrate =. ;
```

STEP 1: Simultaneously estimate va and siga

```
proc model data= indata MAXERRORS=1 noprint ;
by gvkey year ;
bounds 0 < va  siga;
eq.call = va * exp( -divrate * time )*probnorm(((log( va / x )
    + time*( r - divrate + siga * siga / 2 ))) / ( siga * sqrt(time)))
    - x * exp( -r * time ) * probnorm((( log( va / x )
    + time*( r - divrate - siga * siga / 2))) / ( siga * sqrt(time)))
    + (1 - exp( -divrate * time )) * va - ve ;
eq.hedge = ( siga * va * exp( -divrate * time ) / ve ) * probnorm((( log(va / x)
    + time * ( r - divrate + siga * siga / 2 ))) / ( siga * sqrt(time))) - sige ;
solve va siga
/ out = bsmdata  maxiter =50 maxsubit =20 ;
id gvkey year r data19 data21 divrate x ve sige va siga ;
run ;
```

STEP 2: Compute mu

```
proc sort data=bsmdata ;
by gvkey year ;
run ;
data bsmdata ;
set bsmdata ;
     gvkeylag = lag(gvkey) ;
     yearlag = lag(year) ;
     tempvar = lag(va) ;
     if (gvkey = gvkeylag) and (year = yearlag + 1) then valag = tempvar ;
     else valag = . ;
```

```
mu = (va + data19 + data21 - valag) / valag ;
if mu < r then mu = r ;
if mu >1 then mu =1 ;
if valag = . then mu = . ;
```

STEP 3: Compute the probability of bankruptcy, bsmprob

```
if _errors_ = 0 then bsmtemp1 = ((log(va / x)
+ time * ( mu - divrate - siga * siga / 2))) / (siga * sqrt(time));
if _errors_ = 0 then bsmprob = 1-probnorm(bsmtemp1) ;
```

STEP 4: To compute and winsorize bsmscore

```
bsmtemp2 = bsmprob / (1 - bsmprob) ;
bsmscore = log(bsmtemp2) ;
if bsmprob < 0.00001 then bsmscore = -11.51292 ;
if bsmprob > 0.99999 then bsmscore =  11.51292 ;
if bsmprob = . then bsmscore = .;
drop  _type_  _mode_  _errors_  tempvar bsmtemp1 bsmtemp1 ;
run ;
```

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Notes

[1]Studies that utilize the *Z-Score* and/or the *O-Score* include Begley, Ming, and Watts (1996), Berger, Ofek, and Swary (1996), Burgstahler, Jiambalvo, and Noreen (1989), Dichev (1998), Francis (1990), Griffin and Lemmon (2002), Han, Jennings, and Noel (1992), Stone (1991), and Subramanyam and Wild (1996).

[2]In addition to four accounting ratios, the *Z-Score* includes the market value of equity to total liabilities ratio (V_E/TL). For expositional purposes, we refer to the *Z-Score* as "accounting-based."

[3]This omission also affects other studies, including Billings (1999), Dhaliwal, Lee, and Fargher (1992), Opler and Titman (1994), that use leverage as a PB proxy. See Lys (1984) for a setting where volatility is an important correlated omitted variable when only a leverage ratio is used to proxy for financial risk.

[4]Our results show that parameter stability in the accounting-based models is relatively low as several coefficients actually change in sign and magnitude over time.

[5]Bankruptcy can only occur at time T because BSM assumes that the firm issues only zero-coupon bonds. Subsequent studies have incorporated more realistic assumptions, such as allowing for debt covenants (Black and Cox (1976)) and multiple classes of debt (Geske (1977)). We leave an evaluation of alternative option pricing models to future research.

[6]Several commercial vendors offer default probabilities based on option pricing models, with KMV, LLC being the most well-known. We discuss the major differences between our approach and that taken by KMV below.

[7]We use the approximate market value of assets to compute δ because δ is used to estimate V_A. Observations where δ is either negative or greater than 100% are excluded from the analysis.

[8]We also cap the maximum growth rate at 100%. One could use a more sophisticated method to estimate the expected growth rate in asset values. While somewhat *ad hoc*, the current procedure has the advantage of being easy to implement and does not lead to a reduction in our sample size. Additionally, unreported results indicate that the results reported below are not sensitive to the exact method of calculating μ.

[9]With the exception of the sales-to-total assets ratio (S/TA), we multiply the original Altman (1968) coefficients by 100. This adjustment is necessary because in the original study, each independent variable aside from S/TA was expressed as a percentage rather than in ratio form.

[10]We will provide a copy of the program that runs the logit-based Vuong test on request.

[11]While both the discrete hazard model and the method used to estimate the updated *Z-Score*^{u} and *O-Score*^{u} coefficients incorporate multiple firm-year observations, the latter method excludes a proxy for the baseline hazard rate. We do not include a hazard rate variable because it is not included in the original models.

[12]The leverage ratio in the Altman (1968) model is based on the market value of equity, while the ratio in *BSM-Prob* is based on the market value of assets. The (untabulated) correlation between the two leverage ratios is 0.99.

[13]As a robustness check, we used industry classifications based on two-digit SIC codes, Sharpe (1982), and Barth, Beaver, and Landsman (1998). Unreported Vuong tests confirm that *BSM-Score* alone continues to provide significantly more information than the industry-adjusted accounting variables in each specification.

References

Altman, E. (1968): "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," *Journal of Finance*, 23, 589–609.

Barth, M., W. Beaver, and W. Landsman (1998): "Relative Valuation Roles of Equity Book Value and Net Income as a Function of Financial Health," *Journal of Accounting and Economics*, 25, 1–34.

Beaver, W. (1966): "Financial Ratios as Predictors of Bankruptcy," *Journal of Accounting Research*, 6, 71–102.

——— (1968): "Market Prices, Financial Ratios, and the Prediction of Failure," *Journal of Accounting Research*, 8, 179–92.

Beck, N., J. Katz, and R. Tucker (1998): "Taking Time Seriously: Time-Series-Cross-Section Analysis with a Binary Dependent Variable," *American Journal of Political Science*, 42, 1260–1288.

Begley, J., J. Ming, and S. Watts (1996): "Bankruptcy Classification Errors in the 1980's: An Empirical Analysis of Altman's and Ohlson's Models," *Review of Accounting Studies*, 1, 267–284.

Berger, P., E. Ofek, and I. Swary (1996): "Investor Valuation of the Abandonment Option," *Journal of Financial Economics*, 42, 257–287.

Billings, B. (1999): "Revisiting the Relation Between the Default Risk of Debt and the Earnings Response Coefficient," *Accounting Review*, 74(4), 509–522.

Black, F., and J. Cox (1976): "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions," *Journal of Finance*, 31(2), 351–367.

Black, F., and M. Scholes (1973): "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 7, 637–54.

Burgstahler, D., J. Jiambalvo, and E. Noreen (1989): "Changes in the Probability of Bankruptcy and Equity Value," *Journal of Accounting and Economics*, 11, 207–24.

Campbell, J., M. Lettau, B. Malkiel, and Y. Xu (2001): "Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk," *Journal of Finance*, 56(1), 1–43.

Cheung, J. (1991): "A Review of Option-Pricing Theory in Accounting Research," *Journal of Accounting Literature*, 10, 51–84.

Core, J., and C. Schrand (1999): "The Effect of Accounting-Based Debt Covenants on Equity Valuation," *Journal of Accounting and Economics*, 27(1), 1–34.

Dambolena, I., and S. Khoury (1980): "Ratio Stability and Corporate Failure," *Journal of Finance*, 35(4), 1017–26.

Delaney, K. (1992): *Strategic Bankruptcy*. University of California Press, Berkeley, CA.

Dhaliwal, D., K. Lee, and N. Fargher (1992): "The Association Between Unexpected Earnings and Abnormal Security Returns in the Presence of Financial Leverage," *Contemporary Accounting Research*, 8, 20–41.

Dhaliwal, D., and S. Reynolds (1994): "The Effect of the Default Risk of Debt on the Earnings Response Coefficient," *Accountng Review*, 69(2), 412–419.

Dichev, I. (1998): "Is the Risk of Bankruptcy a Systematic Risk?," *Journal of Finance*, 53(3), 1131–47.

Fama, E., and K. French (1997): "Industry Costs of Equity," *Journal of Financial Economics*, 43(2), 153–193.

Francis, J. (1990): "Corporate Compliance with Debt Covenants," *Journal of Accounting Research*, 28, 326–347.

Geske, R. (1977): "The Valuation of Corporate Liabilities as Compound Options," *Journal of Financial and Quantitative Analysis*, pp. 541–552.

Griffin, J., and M. Lemmon (2002): "Book-to-Market Equity, Distress Risk, and Stock Returns," *Journal of Finance*, 57(5), 2317–2336.

Han, B., R. Jennings, and J. Noel (1992): "Communication of Nonearnings Information at the Financial Statements Release Date," *Journal of Accounting and Economics*, 15, 63–86.

Lo, A. (1986): "Logit versus Discriminant Analysis: A Specification Test and Application to Corporate Bankruptcies," *Journal of Econometrics*, 31, 151–78.

Lys, T. (1984): "Mandated Accounting Changes and Debt Covenants: The Case of Oil and Gas Accounting," *Journal of Accounting and Economics*, 7, 39–65.

McDonald, R. (2002): *Derivaties Markets*. first edn.

McFadden, D. (1976): "A Comment on Discrimimant Analysis versus Logit Analysis," *Annals of Economic and Social Measurement*, 5, 511–523.

Merton, R. (1974): "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29, 449–70.

Ohlson, J. (1980): "Financial Ratios and the Probabilistic Prediction of Bankruptcy," *Journal of Accounting Research*, 19, 109–131.

Opler, T., and S. Titman (1994): "Financial Distress and Corporate Performance," *Journal of Finance*, 49(3), 1015–1040.

Palepu, K. (1986): "Predicting Takeover Targets: A Methodological and Empirical Analysis," *Journal of Accounting and Economics*, 8, 3–35.

Platt, H., and M. Platt (1991): "A Note on the Use of Industry-Relative Ratios in Bankruptcy Prediction," *Journal of Banking and Finance*, 15, 1183–1194.

Rogers, W. (1993): "Regression Standard Errors in Clustered Samples," *Stata Technical Bulletin*, 13, 88–94.

Sharpe, W. (1982): "Factors in New York Stock Exchange Security Returns," *Journal of Portfolio Management*, 8(4), 5–19.

Shumway, T. (2001): "Forecasting Bankruptcy More Accurately: A Simple Hazard Model," *Journal of Business*, 74(1), 101–124.

Stone, M. (1991): "Firm Financial Stress and Pension Plan Continuation/Replacement Decisions," *Journal of Accounting and Public Policy*, 10, 175–206.

Subramanyam, K., and J. Wild (1996): "The Going Concern Assumption and the Informativeness of Earnings," *Contemporary Accounting Research*, 13, 251–274.

Vassalou, M., and Y. Xing (2003): "Default Risk in Equity Returns," *Journal of Finance, forthcoming.*

Vuong, Q. (1989): "Likelihood Ratio Tests for Model Selection and Non-Nested Hypotheses," *Econometrica*, 57(2), 307–333.

Warren, S. (2003): "Asbestos Quagmire," *Wall Street Journal*, Jan. 27, B1.

Zmijewski, M. (1984): "Methodological Issues Related to the Estimation of Financial Distress Prediction Models," *Journal of Accounting Research*, Supplement, 59–86.

Table 1: Sample Description

Panel A: Bankruptcies by Year

Year	Number of Firms	Number of Bankruptcies	% of Bankrupt Firms
1980	3,034	21	0.69%
1981	3,138	25	0.80%
1982	3,239	17	0.52%
1983	3,331	39	1.17%
1984	3,492	46	1.32%
1985	3,472	29	0.84%
1986	3,513	29	0.83%
1987	3,660	35	0.96%
1988	3,596	34	0.95%
1989	3,489	44	1.26%
1990	3,469	32	0.92%
1991	3,422	18	0.53%
1992	3,615	23	0.64%
1993	3,780	18	0.48%
1994	4,089	23	0.56%
1995	4,237	23	0.54%
1996	4,426	29	0.66%
1997	4,608	54	1.17%
1998	4,437	51	1.15%
1999	4,137	78	1.89%
2000	3,916	88	2.25%
Total	78,100	756	0.97%

Panel B: Bankruptcies by Industry - Fama and French (1997) Classifications

Industry	Number of Firm-Years	Number of Firms	Bankruptcies	% of Firm-Years Bankrupt	% of Firms Bankrupt
Aerospace	13	57	2	15.38%	3.51%
Agriculture	293	70	0	0.00%	0.00%
Automobiles	1,402	206	17	1.21%	8.25%
Beer	264	50	0	0.00%	0.00%
Books	906	130	5	0.55%	3.85%
Boxes	320	52	1	0.31%	1.92%
Building Materials	2,395	351	14	0.58%	3.99%
Business Services	6,863	2,027	38	0.55%	1.87%
Chemicals	1,586	216	5	0.32%	2.31%
Chips	4,884	792	24	0.49%	3.03%
Clothing	1,387	236	20	1.44%	8.47%
Coal	95	26	5	5.26%	19.23%
Computers	3,625	805	60	1.66%	7.45%
Construction	828	225	18	2.17%	8.00%
Drugs	2,808	573	5	0.18%	0.87%
Elec. Equipment	1,466	197	12	0.82%	6.09%
Energy	4,232	803	58	1.37%	7.22%
Fabricated Products	519	72	4	0.77%	5.56%
Food	1,775	275	11	0.62%	4.00%
Gold	577	99	7	1.21%	7.07%
Guns	78	15	0	0.00%	0.00%
Health	1,485	342	22	1.48%	6.43%
Households	2,005	299	18	0.90%	6.02%
Lab Equipment	2,060	295	7	0.34%	2.37%
Leisure	1,367	345	15	1.10%	4.35%
Machinery	3,477	503	24	0.69%	4.77%
Medical Equipment	2,480	483	7	0.28%	1.45%
Mines	438	68	2	0.46%	2.94%
Miscellaneous	924	189	6	0.65%	3.17%
Non-Beer Beverages	145	31	0	0.00%	0.00%
Paper	1,632	191	11	0.67%	5.76%
Personal Services	720	157	10	1.39%	6.37%
Restaurants	1,837	355	25	1.36%	7.04%
Retail	4,714	915	105	2.23%	11.48%
Rubber	1,270	186	8	0.63%	4.30%
Shipping	186	34	3	1.61%	8.82%
Steel	1,626	209	18	1.11%	8.61%
Telecommunications	2,207	578	33	1.50%	5.71%
Textiles	836	120	19	2.27%	15.83%
Tobacco	89	20	0	0.00%	0.00%
Toys	924	175	18	1.95%	10.29%
Transportation	2,142	399	31	1.45%	7.77%
Utilities	4,617	428	17	0.37%	3.97%
Wholesalers	4,161	704	51	1.23%	7.24%
Total	77,658	14,303	756	0.97%	5.29%

Table 2: Original and Updated Coefficients for Altman (1968) and Ohlson (1980)

Altman (1968) Model:	WC/TA	RE/TA	EBIT/TA	V_E/TL	S/TA	Constant
Original Coefficients[a]	-1.20	-1.40	-3.30	-0.60	-0.999	
Updated Coefficients[b]	-0.08	0.04	-0.10**	-0.22***	0.06	-4.34***

Ohlson (1980) Model I:	Size	TL/TA	WC/TA	CL/CA	NI/TA	FFO/TL	INTWO	OENEG	CHIN	Constant
Original Coefficients	-0.41***	6.03***	-1.43**	0.08	-2.37**	-1.83***	0.285	-1.72***	-0.52***	-1.32
Updated Coefficients[c]	0.04***	0.08***	0.01***	-0.01	1.20**	0.18***	0.01***	1.59***	-1.10***	-5.91***

*** (**) [*] significant at the 1% (5%) [10%] level (two-sided test)

[a] The signs of the original coefficients have been changed so the Z-Score is increasing in the probability of bankruptcy. All of the coefficients except S/TA have been multiplied by 100 since, unlike Altman (1968), we define all of our variables as ratios. Altman (1968) does not report significance levels.

[b] Updated coefficients are estimated in a logit regression that includes all available firm-years. The sample consists of 89,826 firm-year observations including 762 initial bankruptcies between 1980 and 2000. The dependent variable is bankruptcy in the 4 to 16 months following the fiscal year-end.

[c] Updated coefficients are estimated in a logit regression that includes all available firm-years. The sample consists of 89,643 firm-year observations including 809 initial bankruptcies between 1980 and 2000. The dependent variable is bankruptcy in the 4 to 16 months following the fiscal year-end.

The dependent variable is bankruptcy in the 4 to 16 months following the fiscal year-end.

WC/TA is working capital divided by total assets.

RE/TA is retained earnings divided by total assets.

EBIT/TA is earnings before interest and taxes divided by total assets.

V_E /TL is market value of equity divided by total liabilities.

S/TA is sales divided by total assets.

Size is the ln(Total Assets/GDP price level index).

TL/TA is total liabilities divided by total assets.

CL/CA is current liabilities divided by current assets.

NI/TA is net income divided by total assets.

FFO/TL is pre-tax income plus depreciation and amortization divided by total liabilities.

INTWO is an indicator variable equal to one if cumulative net income over the previous two years is negative, and zero otherwise.

OENEG is an indicator variable equal to one if owners' equity is negative, and zero otherwise.

CHIN = $(NI_t - NI_{t-1})/(|NI_t| + |NI_{t-1}|)$ is the scaled change in net income.

Table 3: Descriptive Statistics

Based on 78,100 firm-years including 756 initial bankruptcies.

Variable	Status	Mean		Std Dev	Median		1% Percentile	99% Percentile
AnnualRate	Solvent	0.87%		0.39%	0.79%		0.41%	2.18%
	Bankrupt	1.05%	***	0.48%	0.99%	***	0.41%	2.18%
BSM-Prob	Solvent	5.61%		15.31%	0.01%		0.00%	84.16%
	Bankrupt	24.76%	***	26.84%	15.47%	***	0.00%	98.04%
Z-Prob	Solvent	13.46%		21.97%	4.63%		0.00%	99.97%
	Bankrupt	39.21%	***	32.18%	28.86%	***	0.03%	100.00%
Z-Prob^u	Solvent	0.77%		1.23%	0.79%		0.00%	1.52%
	Bankrupt	1.21%	***	1.14%	1.20%	***	0.05%	4.18%
O-Prob	Solvent	29.35%		31.93%	14.95%		0.05%	100.00%
	Bankrupt	71.61%	***	29.00%	82.90%	***	3.47%	100.00%
O-Prob^u	Solvent	0.91%		1.73%	0.37%		0.11%	7.14%
	Bankrupt	2.66%	***	2.94%	1.70%	***	0.12%	12.97%

*** (**) [*] The bankrupt firm-years are significantly different from the solvent firm-years at the 1% (5%) [10%] level (two-sided test) based on a t-test for the test of means and the Wilcoxon rank-sum test for the test of medians.

AnnualRate is the economy-wide percentage rate of corporate bankruptcies among publicly-traded firms over the previous twelve months.

BSM-Prob is the estimated probability of bankruptcy based on the Black-Scholes-Merton model.

Z-Prob is derived from the variables and coefficient estimates of Altman (1968). The Z-Score is converted into a probability using the logistic function.

Z-Prob^u is the probability of bankruptcy based on the variables in Altman (1968). Updated coefficients are estimated in a logit regression including multiple firm-year observations based on an expanding rolling window approach.

O-Prob is the probability of bankruptcy derived from the variables and coefficient estimates of Ohlson (1980).

O-Prob^u is the probability of bankruptcy based on the variables in Ohlson (1980). Updated coefficients are estimated in a logit regression including multiple firm-year observations based on an expanding rolling window approach.

Table 4: Correlation Matrix

Based on 78,100 non-bankrupt firm-years including 756 initial bankruptcies.
Pearson correlations are below the diagonal and Spearman correlations are above the diagonal.

Variable	Bankrupt	AnnualRate	BSM-Prob	Z-Prob	Z-Prob^u	O-Prob	O-Prob^u
Bankrupt		0.04	0.11	0.10	0.09	0.11	0.09
AnnualRate	0.05		0.08	0.07	0.11	0.03	0.08
BSM-Prob	0.12	0.09		0.43	0.34	0.49	0.33
Z-Prob	0.11	0.09	0.43		0.63	0.63	0.36
Z-Prob^u	0.03	0.03	0.10	0.16		0.40	0.36
O-Prob	0.13	0.03	0.39	0.65	0.12		0.20
O-Prob^u	0.10	0.03	0.21	0.41	0.33	0.49	

All correlations are significant at the 1% level (two-sided test).

Bankrupt is an indicator variable that equals one if the firm declares bankruptcy in the four to sixteen months following the firm's fiscal year end, and zero otherwise.

AnnualRate is the economy-wide percentage rate of corporate bankruptcies among publicly-traded firms over the previous twelve months.

BSM-Prob is the estimated probability of bankruptcy based on the Black-Scholes-Merton model.

Z-Prob is derived from the variables and coefficient estimates of Altman (1968). The Z-Score is converted into a probability using the logistic function.

Z-Prob^u is the fitted probability of bankruptcy based on the variables in Altman (1968). Updated coefficients are estimated in a logit regression including multiple firm-year observations based on an expanding rolling window approach.

O-Prob is the probability of bankruptcy derived from the variables and coefficient estimates of Ohlson (1980).

O-Prob^u is the probability of bankruptcy based on the variables in Ohlson (1980). Updated coefficients are estimated in a logit regression including multiple firm-year observations based on an expanding rolling window approach.

Table 5: Relative Information Content of Bankruptcy Scores

Discrete Hazard Regression of Bankruptcy Scores on Bankruptcies the Following Year

	Z-Score	Z-Score^{u}	O-Score	O-Score^{u}	BSM-Score
Constant	-4.95***	-1.12***	-5.44***	-1.47***	-3.77***
AnnualRate	0.75***	0.81***	0.83***	0.79***	0.54***
Z-Score	0.18***				
Z-Score^{u}		0.88***			
O-Score			0.21***		
O-Score^{u}				0.82***	
BSM-Score					0.27***
Log Likelihood	-3,972	-4,024	-3,881	-3,831	-3,728
Pseudo-R^2	0.07	0.06	0.09	0.10	0.12
Observations	78,100	78,100	78,100	78,100	78,100

*** $^{(**)}$ $^{[*]}$ significant at the 1% (5%) [10%] level (two-sided test)

p-values are based on Huber-White standard errors.

The dependent variable is an indicator variable that equals one if the firm declares bankruptcy in the four to sixteen months following the firm's fiscal year end, and zero otherwise.

AnnualRate is the economy-wide percentage rate of corporate bankruptcies among publicly-traded firms over the previous twelve months.

BSM-Score is the bankruptcy score based on the Black-Scholes-Merton model where BSM-Prob is transformed into a bankruptcy score using the inverse logistic function.

Z-Score is derived from the variables and coefficient estimates of Altman (1968).

Z-Score^{u} is the bankruptcy score based on the variables in Altman (1968). Updated coefficients are estimated in a logit regression including multiple firm-year observations based on an expanding rolling window approach.

O-Score is derived from the variables and coefficient estimates of Ohlson (1980).

O-Score^{u} is the bankruptcy score based on the variables in Ohlson (1980). Updated coefficients are estimated in a logit regression including multiple firm-year observations based on an expanding rolling window approach.

Score variables are winsorized so that the minimum (maximum) value is -11.51292 (+11.51292).

Table 6: Decomposition of Bankruptcy Scores into Industry Means and Deviations

Discrete Hazard Regression of Decomposed Bankruptcy Scores on Bankruptcies the Following Year

Panel A: Industry Means and Deviations

	Z-Score	Z-Score^u	O-Score	O-Score^u	BSM-Score
Constant	-5.01***	-0.46***	-5.75***	-3.88***	-4.51***
AnnualRate	0.76***	0.78***	0.89***	0.89***	0.68***
Z-Score$_{IM}$	0.17***				
Z-Score$_{ID}$	0.18***				
Z-Score$^u{}_{IM}$		1.00***			
Z-Score$^u{}_{ID}$		0.86***			
O-Score$_{IM}$			0.02		
O-Score$_{ID}$			0.22***		
O-Score$^u{}_{IM}$				0.40**	
O-Score$^u{}_{ID}$				0.84***	
BSM-Score$_{IM}$					0.19***
BSM-Score$_{ID}$					0.28***
Log Likelihood	-3,972	-4,022	-3,872	-3,823	-3,722
Pseudo-R^2	0.07	0.06	0.09	0.10	0.13
Observations	78,100	78,100	78,100	78,100	78,100

IM is the prior year's industry mean of the variable based on the Fama and French (1997) industry classification system.
ID is the variable's deviation from the prior year's industry mean.

Panel B: Vuong Tests

	***z*-statistic**
Z-Score - industry mean and deviation versus BSM-Score:	-9.31***
Z-Score^u - industry mean and deviation versus BSM-Score:	-9.91***
O-Score - industry mean and deviation versus BSM-Score:	-5.32***
O-Score^u - industry mean and deviation versus BSM-Score:	-2.88***

*** (**) [*] significant at the 1% (5%) [10%] level (two-sided test).

p-values are based on Huber-White standard errors.

Negative (Positive) *z*-statistic indicates that BSM-Score provides relatively more (less) information.

Table 7: Decomposition of Bankruptcy Scores into Lagged Levels and Changes

Discrete Hazard Regression of Decomposed Bankruptcy Scores on Bankruptcies the Following Year

Panel A: Levels and Changes Models

	Z-Score	Z-Score^u	O-Score	O-Score^u	BSM-Score
Constant	-4.96***	0.37	-5.40***	-0.92***	-3.87***
AnnualRate	0.74***	0.73***	0.82***	0.78***	0.54***
Z-Score$_{t-1}$	0.18***				
ΔZ-Score	0.20***				
Z-Score$^u_{t-1}$		1.19***			
ΔZ-Score^u		1.32***			
O-Score$_{t-1}$			0.22***		
ΔO-Score			0.23***		
O-Score$^u_{t-1}$				0.92***	
ΔO-Score^u				0.84***	
BSM-Score$_{t-1}$					0.26***
ΔBSM-Score					0.29***
Log Likelihood	-3,356	-3,379	-3,267	-3,213	-3,137
Pseudo-R^2	0.07	0.06	0.09	0.11	0.13
Observations	65,627	65,627	65,627	65,627	65,627

t-1 signifies the level of the variable at the beginning of year t-1 .

Δ signifies the change in variable over year t-1.

Panel B: Vuong Tests

	z-statistic
Z-Score - lagged level and change versus BSM-Score alone:	-8.77***
Z-Score^u - lagged level and change versus BSM-Score alone:	-8.07***
O-Score - lagged level and change versus BSM-Score alone:	-5.21***
O-Score^u - lagged level and change versus BSM-Score alone:	-2.44**

*** (**) [*] significant at the 1% (5%) [10%] level (two-sided test).

p-values are based on Huber-White standard errors.

Negative (Positive) *z*-statistic indicates that BSM-Score provides relatively more (less) information.

Table 8: Incremental Information Content of Accounting-based Bankruptcy Scores

Discrete Hazard Regression of Multiple Bankruptcy Scores on Bankruptcies the Following Year.

Variable	Column 1	Column 2	Column 3	Column 4
Constant	-3.76***	-1.51***	-4.10***	-1.45***
AnnualRate	0.51***	0.53***	0.57***	0.52***
BSM-Score	0.23***	0.24***	0.22***	0.21***
Z-Score	0.09***			
Z-Score^u		0.49***		
O-Score			0.13***	
O-Score^u				0.55***
Log Likelihood	-3,692	-3,679	-3,636	-3,592
Pseudo-R^2	0.13	0.14	0.15	0.16
Observations	78,100	78,100	78,100	78,100

*** (**) [*] significant at the 1% (5%) [10%] level (two-sided test)

p-values are based on Huber-White standard errors.

Bankrupt is an indicator variable that equals one if the firm declares bankruptcy in the four to sixteen months following the firm's fiscal year end, and zero otherwise.

AnnualRate is the economy-wide rate of corporate bankruptcies among publicly-traded firms over the previous twelve months.

BSM-Score is the bankruptcy score based on the Black-Scholes-Merton model where BSM-Prob is transformed into a bankruptcy score using the inverse logistic function.

Z-Score is derived from the variables and coefficient estimates of Altman (1968).

Z-Score^u is the bankruptcy score based on the variables in Altman (1968). Updated coefficients are estimated in a logit regression including multiple firm-year observations based on an expanding rolling window approach.

O-Score is derived from the variables and coefficient estimates of Ohlson (1980).

O-Score^u is the bankruptcy score based on the variables in Ohlson (1980). Updated coefficients are estimated in a logit regression including multiple firm-year observations based on an expanding rolling window approach.