Choosing Bankruptcy Predictors Using Discriminant Analysis, Logit Analysis, and Genetic Algorithms

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Abstract

We are focusing on three alternative techniques that can be used to empirically select predictors for failure prediction purposes. The selected techniques have all different assumptions about the relationships between the independent variables. Linear discriminant analysis is based on linear combination of independent variables, logit analysis uses the logistic cumulative probability function and genetic algorithms is a global search procedure based on the mechanics of natural selection and natural genetics. Our aim is to study if these essential differences between the methods (1) affect the empirical selection of independent variables to the model and (2) lead to significant differences in failure prediction accuracy.

Keywords: Bankruptcies, genetic algorithms, neural networks.

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1. Introduction

Bankruptcy prediction has been one of the most challenging tasks in accounting since the study of Fitzpatrick in 1930's and during the last 60 years an impressive body of theoretical and especially empirical research concerning this topic has evolved (for reviews see Zavgren 1983, Altman 1983, Jones 1987). Two main approaches in bankruptcy prediction studies can be distinguished: The first and most often used approach has been the empirical search for predictors (financial ratios) that lead to lowest misclassification rates. The second approach has concentrated on the search for statistical methods that would also lead to improved prediction accuracy.

At the beginning of the research period of failure prediction (see e.g. Fitzpatrick, 1932) there were no advanced statistical methods or computers available for the researchers. The values of financial ratios in failed and nonfailed firms were compared with each other and it was found that they were poorer for failed firms. In 1966 the pioneering study of Beaver presented the univariate approach of discriminant analysis and in 1968 Altman expanded this analysis to multivariate analysis. Until 1980's discriminant analysis was the dominant method in failure prediction. However, it suffered from assumptions that were violated very often. The assumption of normality of the financial ratio distributions was problematic, particularly for the failing firms. During the 1980's the method was replaced by logistic analysis which until last years has been the most used statistical method for failure prediction purposes.

During the 1990's artificial neural networks have produced very promising results in predicting bankruptcies (Wilson & Sharda, 1995, Serrano-Cinca, 1993 and Back et al, 1994). However, no systematic way of identifying the predictive variables for the neural networks has been used in these studies. Genetic algorithms are a new promising method for finding the best set of indicators for neural networks. These algorithms have been applied successfully in several optimisation problems, especially in technical fields.

Most failure prediction studies (done before 1980's) applied an empirical approach, i.e., they aimed at improved prediction accuracy by appropriate selection of financial ratios for the analysis. Naturally, these financial ratios have been selected according to their ability to increase prediction accuracy. There are some efforts to create theoretical constructions in failure prediction context (for presentation, see e.g. Scott 1981), but none unified theory has been generally accepted as a basis for the theoretical ratio selection. Hence, the selection has been based on the empirical characteristics of the ratios. This has led to a research tradition in which also the effect of statistical method on predictor selection has been obvious. This is because e.g. the stepwise selection procedures identify variables solely on statistical grounds, ignoring the other characteristics of the variable.

Discriminant analysis, logit analysis and genetic algorithms have all different assumptions concerning the relationships between the independent variables. Linear discriminant analysis is based on linear combination of independent variables, logit analysis uses the logistic cumulative probability function and genetic algorithm is a global procedure based on the mechanics of natural selection and natural genetics. Our aim is to study if these essential differences between the methods (1) affect in the first phase the empirical selection of independent variables to the prediction model and (2) lead to significant differences in failure prediction accuracy.

We have taken eleven central studies concerning the financial distress of a firm as a starting point. From this literature we have selected 31 financial ratios which in previous studies have been found useful in bankruptcy prediction. Using the data consisting of 37 matched pairs we examine which of these ratios will become important predictors of failure when alternative techniques are applied. SAS stepwise procedure is used to select predictors for discriminant analysis, which has been the method applied in all eleven original studies. After that we apply logit analysis to find out if the set of predictors is changed compared with those of discriminant analysis. Furthermore, we use genetic algorithms for finding the best set of predictors for neural networks. At the final stage we compare the three groups of selected ratios, and the prediction accuracy results achieved by the three different methods, i.e. discriminant analysis, logit analysis, and neural networks.

The rest of the paper is organised as follows. In next section we give a short description of the three different techniques and their variable selection methods. In section 3, we present the criteria for and the choice of the 31 ratios and the data. Section 4-7 present the empirical results and section 8 concludes the paper.

2 Three alternative variable selection and prediction techniques

Discriminant analysis Discriminant analysis tries to derive the linear combination of two or more independent variables that will discriminate best between a priori defined groups, which in our case are failing and non-failing companies. This is achieved by the statistical decision rule of maximising the between-group variance relative to the within group variance. This relationship is expressed as the ratio of between-group to within-group variance. The discriminant analysis derives the linear combinations from an equation that takes the following form:

$$Z = w_1x_1 + w_2x_2 + \ldots + w_nx_n$$

where

Z	= discriminant score
w_i (i=1, 2, ... ,n)	= discriminant weights
x_i (i=1, 2, ... ,n)	= independent variables, the financial ratios

Thus, each firm receives a single composite discriminant score which is then compared to a cut-off value, which determines to which group the company belongs to.

Discriminant analysis does very well provided that the variables in every group follow a multivariate normal distribution and the covariance matrices for every group are equal. However, empirical experiments have shown that especially failing firms violate the normality condition. In addition, the equal group variances condition is also violated. Moreover, multicollinearity among independent variables is often a serious problem, especially when stepwise procedures are employed (Hair et al., 1992). However, empirical studies have proved that the problems connected with normality assumptions were not weakening its classification capability, but its prediction ability.

The two most frequently used methods in deriving the discriminant models have been the *simultaneous (direct) method* and the *stepwise method.* The former is based on model construction by e.g. theoretical grounds, so that the model is ex ante defined and then used in discriminant analysis. When the stepwise method is applied, the procedure selects a subset of variables to produce a good discrimination model using forward selection, backward elimination, or stepwise selection. The stepwise method that we used is a built in function in the SAS-program.

The stepwise selection begins with no variables in the model. At each step, if the variable that contributes least to the discriminatory power of the model measured by Wilks` lambda fails to meet the criterion to stay, then it will be removed. The variable not in the model that contributes most to the discriminatory power of the model is entered. When all variables in the model meet the criterion to stay and none of the other variables meets the criterion to enter, the stepwise selection process stops (SAS 1988:910).

There are also other modelling strategies, like the one suggested by Hosmer and Lemeshow (1989). In some contexts it has been stated that the disadvantage of stepwise procedure is that the economic importance of variables is ignored and the statistical grounds are stressed. The modelling strategy by Hosmer and Lemeshow is less mechanical in that it allows for the analyst's judgement concerning variables to be included in the model.

Logit analysis Logistic regression analysis has also been used to investigate the relationship between binary or ordinal response probability and explanatory variables. The method fits linear logistic regression model for binary or ordinal response data by the method of maximum likelihood. Among the first users of logit analysis in the context of financial distress was Ohlson (1980). Like discriminant analysis, this technique weights the independent variables and assigns a Z score in a form of failure probability to each company in a sample. The advantage of this method is that it does not assume multivariate normality and equal covariance matrices as discriminant analysis does. Logit analysis incorporates non-linear effects, and uses the logistical cumulative function in predicting a bankruptcy, i.e.,

$$\text{Probability of failure} \frac{1}{1 + e^{(-Z)}} = \frac{1}{1 + e^{-(w_0+w_1x_1+...+w_nx_n)}}$$

Logistic analysis applies the same variable selection methods as discriminant analysis presented above. For model construction we selected, as in the case of discriminant analysis, the stepwise method that is a built in function in the SAS-program. The procedure starts by estimating parameters for variables forced into the model, i.e. intercept and the first possible explanatory variables. Next, the procedure computes the adjusted chi-squared statistic for all the variables not in the model and examines the largest of these statistics. If it is significant at the specified level, in our study 0.05, the variable is entered into the model. Each selection step is followed by one or more elimination step, i.e. the variables already selected into the model do not necessarily stay. The stepwise selection process terminates if no further variable can be added to the model, or if the variable just entered into the model is the only variable removed in the subsequent elimination (see SAS 1990).

Neural networks An (artificial) *neural network* consists of a large number of processing elements, *neurons*, and connections between them. It implements some *function* f that maps a set of given input values x to some output values y: $y = f(x)$. A neural network tries to find the best possible approximation of the function f. This approximation is coded in the neurons of the network using *weights* that are associated with each neuron.

A formal *neuron* is the basic element of any neural network. A neuron is a simple processing element that as inputs takes an n-dimensional vector $[x_1, \ldots , x_n]^T$, extended with a constant component $x_0 = 1$. The neuron forms the weighted sum

$$w^T x = w_0 + S_{1 \le i \le n} w_i x_i,$$

where $x = [1, x_1, \ldots , x_n]^T$ and where $w = [w_0, \ldots, w_n]^T$ is the *weight vector* which is stored in the neuron. In the simplest case the output of a neuron is the sign of this expression, $y = \mathrm{sgn}(w^T x)$. Such a neuron can classify *n*-dimensional vectors into two different classes when the weights are determined so that $y = 1$ for class 1 vectors and $y = -1$ for class 2 vectors.

The weights of a neural network are *learned* using an iterative procedure during which examples of correct input-output associations are shown to the network and the weights get modified so that the network starts to mimic this desirable input-output behaviour. Learning in a neural network then means finding an appropriate set of weights. This ability to learn from examples and based on this learning the ability to generalise to new situations is the most attractive feature of the neural network paradigm. For a more thorough description of neural networks we refer to Hecht-Nielsen (1991), and Hertz et al. (1991).

The variables for the input vectors can be chosen by an exhaustive search from the available variables, but this becomes very time consuming when the choice is to be done among several variables. Another method to choose the variables for the networks is to use a genetic algorithm.

A genetic algorithm simulates Darwinian evolution. It maintains *a population* of chromosomes, where a *chromosome* is a candidate-solution to the problem we want to solve. Chromosomes are often called *strings* in a genetic algorithm context. A string in its turn, consists of a number of *genes*, which may take some number of values, called *alleles*. The genetic algorithm terms for genes and alleles are *features* and *values*. Associated with each string is a *fitness value*, which determines how 'good' a string is. The fitness value is determined by a *fitness function*, which we can think of as some measure of profit or goodness that we want to maximise. Basically, there are three operators that lead to good results in a genetic algorithm, namely reproduction, crossover, and mutation.

Reproduction This is a process in which strings are copied onto the next generation. Strings with a higher fitness value have more chance of making it to the next generation. Different schemes can be used to determine which strings survive into the next generation. A frequently used method is *roulette wheel selection*, where a roulette wheel is divided in a number of slots, one for each string. The slots are sized according to the fitness of the strings. Hence, when we spin the wheel, the best strings are the most likely to be selected. Another well known method is *ranking*. Here, the strings are sorted by their fitness value, and each string is assigned an offspring count that is determined solely by its rank.

Crossover A part of one string is combined with a part of another string. This way, we hope to combine the good parts of one string with the good parts of another string, yielding an even better string after the operation. This operation takes two strings, the parents, and produces two new ones, the offspring. Many kinds of crossover can be thought of. Two kinds of crossover are illustrated in Figure 1a and 1b.

Mutation A randomly selected gene in a string takes a new value. The aim of this operator is to introduce new genetic material in the population, or at least prevent the loss of it. Under mutation, a gene can get a value that did not occur in the population before, or that has been lost due to reproduction. Mutation is illustrated in Figure 1c. For a more thorough description of genetic algorithms we refer to Goldberg (1989).

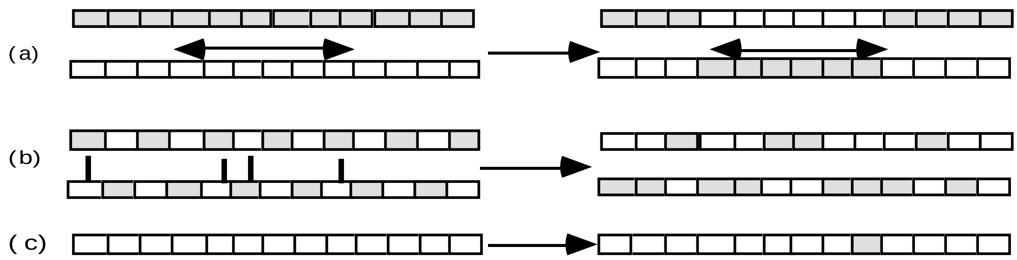

Figure 1. Parts (a) and (b) illustrate two different kinds of crossover operations. Part (c) illustrates a mutation operation.

3 Ratios and data

Ratios Among the studies focusing on failure prediction there are some papers that have been especially contributive. Among the first studies that identify the problemacy connected to the difference between the values of financial ratios of failing and non-failing firms were the studies of Ramser and Foster (1931), Fitzpatrick (1932), Winakor and Smith (1935), and Merwin (1942). These studies settled the fundamentals for failure prediction research.

The change in research tradition took place when Beaver (1966) presented the univariate analysis approach. Soon after Beaver, Altman (1968) pioneered the use of multivariate approach in the context of bankruptcy models. After the Altman study the multivariate approach became dominant in these models. Actually the study of Beaver remained as the main study applying uniavariate analysis. The studies by Deakin (1972), Edminster (1972), Blum (1974), Altman et al. (1977) and El Hennawy and Morris (1983) are representative examples of studies that used multiple discriminant analysis technique. In Table 1 we have collected 31 financial ratios used in the discriminant models in those studies. We have selected in our analysis only those variables that have been found useful in failure prediction.

Data The data comprised the annual financial statements of 37 randomly selected Finnish failed companies and their non-failed mates. Each failure occurred between 1986 and 1989. The time period was not the same for each firm, but the financial statements of matched pairs are always from the same calendar years. The firms in the sample represent different industries, most of them operating in manufacturing (see Table 2). Furthermore, the sample consisted mainly of small and medium-sized companies. The lack of larger companies was explained by the fact that the number of large firms which failed was very small in Finland. In year 1988, for instance, only three firms employing more than 200 people failed (Bankruptcy Bulletin 2/1989).

Table 1. Financial ratios found to be useful in previous bankruptcy prediction studies

Ratios	Study	
R1 Cash/Current Liabilities	L	E, D
R2 Cash Flow/Current Liabilities	L	E
R3 Cash Flow/Total Assets	L	E-M
R4 Cash Flow/Total Debt	L	Bl, B, D
R5 Cash/Net Sales	L	D
R6 Cash/Total Assets	L	D
R7 Current Assets/Current Liabilities	L	M, B, D, A-H-N
R8 Current Assets/Net Sales	L	D
R9 Current Assets/Total Assets	L	D,E-M
R10 Current Liabilities/Equity	L	E
R11 Equity/Fixed Assets	S	F
R12 Equity/Net Sales	S	R-F, E
R13 Inventory/Net Sales	L	E
R14 Long Term Debt/Equity	S	E-M
R15 MV of Equity/Book Value of Debt	S	A, A-H-N
R16 Total Debt/Equity	S	M
R17 Net Income/Total Assets	P	B, D
R18 Net Quick Assets/Inventory	L	Bl
R19 Net Sales/Total Assets	P	R-F, A
R20 Operating Income/Total Assets	P	A, T, A-H-N
R21 EBIT/Total Interest Payments	L	A-H-N
R22 Quick Assets/Current Liabilities	L	D, E-M
R23 Quick Assets/Net Sales	L	D
R24 Quick Assets/Total Assets	L	D, T, E-M
R25 Rate of Return to Common Stock	P	Bl
R26 Retained Earnings/Total Assets	P	A, A-H-N
R27 Return on Stock	P	F, T
R28 Total Debt/Total Assets	S	B, D
R29 Working Capital/Net sales	L	E, D
R30 Working Capital/Equity	L	T
R31 Working Capital/Total Assets	L	W-S,M,B,A,D

Type : L=liquidity, P=profitability, S=solidity
Legend:

A	Altman 1968
A-H-N	Altman, Haldeman, and Narayanan 1977
B	Beaver 1966
Bl	Blum 1974
D	Deakin 1972
E	Edminster 1972
E-M	El Hennawy and Morris 1983
F	Fitzpatrick 1932
M	Merwin 1942
R-F	Ramser and Foster 1931
W-S	Winakor and Smith 1935

Table 2. The distribution of the firms in the sample by industries

ISIC Code	Industry	Frequency
321	Manufacture of textiles	2
322	Manufacture of wearing apparel	4
324	Footwear except rubber and plastic	5
331	Manufacture of wood products, except furniture	3
332	Manufacture of furniture and fixtures	1
381	Manufacture of fabricated metal products	4
382	Manufacture of machinery	3
511	General house contractors	4
512	Special trade contractors	3
612	Wholesaling of food and beverages	1
617	Other wholesaling proper	2
618	Agency	1
624	Textiles, clothing and footwear shops	2
627	Automobile retailing and service	2
Total		37

4 Selected ratios

Discriminant analysis The set of variables for the discriminant analysis was chosen using stepwise selection. Variables were chosen to enter or leave the model using the significance level of an F test from an analysis of covariance, where the already selected variables act as covariates and the variable under consideration is the dependent variable. In our analysis we selected the significance level for adding or retaining variables in the model to be 0.05.

All the 31 ratios for every firm were calculated and the stepwise selection was done among these variables 1, 2, and 3 years prior to failure. The variables that were selected into the discriminant analysis models are presented in Table 3.

Table 3. Variables selected for discriminant analysis.

1 year prior to failure	2 years prior to failure	3 years prior to failure
R16	R14	R4
R4	R27	R5
R19	R5	R28
R24	R25	
	R11	
	R12	

Logit analysis For the logit analysis we selected the variables using the logistic regression procedures available in SAS. As in discriminant analysis models, we used stepwise model-selection method and the same significance level, 0.05, for adding or retaining variables was also applied. The models selected for logistic analysis 1, 2, and 3 years prior to failure are presented in Table 4.

Table 4. Variables selected for logit analysis.

1 year prior to failure	2 years prior to failure	3 years prior to failure
R4	R14	R4
R24	R27	R5
R28	R5	
	R25	

Genetic algorithms The chromosome syntax we used in our genetic algorithm is very straightforward: if an optimal selection was to be made out of N variables we used bitstrings of length N, in which a **1** at position **a** meant that variable **a** was used in the network represented by the bitstring at hand.

Next, the fitness value for a bitstring **s** is determined by training a neural network defined by string **s** for a number of times. During each training run, the minimal prediction error on the test set was logged. The average value of those minimal errors was then used to compute a fitness value, where strings with a lower error obtained a higher fitness value.

After each string was assigned a fitness value, a new population was created in which the offspring yielded by crossover and mutation replaced the worst strings of the old population. Selection of 'parents' for crossover and mutation was done by means of roulette wheel selection.

Two crossover operators were used. In the first one, two strings **a** and **b** are selected, and two cross sites are selected at random. The first offspring, **a'**, inherits the part between the cross sites form **a**, and the other parts from **b**. The second offspring, **b'**, inherits the part between the cross sites form **b** and the other parts from **a**. This is similar to the crossover illustrated in Figure 1 (a).

The second crossover operator also takes two parents **a** and **b**, which are selected by roulette wheel selection. Next, a random number n [0, [N/2]] is generated. Now, n times a string position **p** is selected at random, and the values of **a** and **b** at position **p** are swapped. This is similar to the crossover illustrated in Figure 1(b).

Only one mutation operator was used. A parent is selected by roulette wheel selection. A string position **p** is selected at random, and the value at position **p** is inverted. This is illustrated in Figure 1 (c).
The variables selected for the neural nets 1, 2, and 3 years prior to failure are presented in Table 5

Table 5. Variables selected using genetic algorithm.

1 year prior to failure	2 years prior to failure	3 years prior to failure
R1	R6	R1
R15	R9	R3
R17	R11	R5
R19	R23	R7
R24	R25	R8
R30	R27	R10
	R30	R14
		R15
		R18
		R19
		R20
		R21
		R22
		R27
		R29

5 Analysing the models

In analysing the variables included in the different models we pay attention to the number of variables included, which if any of the variables are included in all three models, respectively, in two models, and to the characteristics of the variables included.

Number of variables selected Table 3 - Table 5 show that there is a big difference between the methods regarding the number of chosen variables. The stepwise model selection used for the logit model chooses by far the fewest number of variables for all three years. The genetic algorithm compared to the stepwise method for the discriminant analysis model or logit model chooses every year the highest number of variables. Three years prior to failure this difference is at highest when discriminant and logit models include 2 to 3 variables, respectively, and genetic algorithm ends up to 15 variables.

The number of selected variables for discriminant and logit models depends on the significance level applied. When i.e. the significance level was raised up to 0.15 for the stepwise model selection for the discriminant model, the amount of included variables one year prior to failure was six, two years prior to failure eight and three years prior to failure five. This amount is one year prior to failure same as for genetic algorithms and two years prior to failure even higher. The selection process for genetic algorithm is very different from the two other studied methods which both are sensitive to the changes in significance level. Anyhow, for the chosen and generally used significance level, 0.05, the number of variables in the models was every year the lowest for logit model and the highest for genetic algorithm model.

Variables included Table 3 - Table 5 show that there is only one variable - R24 (Quick Assets/Total Assets) - that is included in all three models one year prior to failure. There are two variables - R25 (Rate of Return to Common Stock) and R27 (Return on Stock) that are selected for all three models two years prior to failure and again only one variable - R5 (Cash/Net Sales)- that is selected for all three models three years prior to failure.

We observe that, the variables chosen for the logit analysis for all three years, with one exception only, are a subset of the variables chosen for the discriminant analysis.

Common to the discriminant analysis and genetic algorithm model one year prior to failure are R19 (Net Sales/Total Assets) and R24 (Quick Assets/Total Assets), two years prior to failure R11 (Equity/Fixed Assets), R25 (Rate of Return to Common Stock) and R27 (Return on Stock), and three years prior to failure R5 (Cash/Net Sales).

Interestingly there are no variables that are included in both the genetic algorithm model and the logit model but not in the discriminant model. This holds for all three different years prior to failure. Moreover, we have variables that are unique for one model each year.

We have only three variables - R2 (Cash Flow/Current Liabilities), R26 (Retained Earnings/Total Assets), and R31 (Working Capital/Total Assets) that are not chosen at all.

Characteristics of the variables The models are quite different in comparison with each other. There can be at least two explanations for this result. First, there can be real and significant different characteristics in different firms that can be measured by different financial ratios. Hence, alternative empirical methods use this information in alternative ways and thus all three models differ from each other.

The second possible explanation to this divergence is that because we have selected a large amount of ratios in our original data that was used in model construction, it is quite obvious that there are measures which are highly correlated. If the ratios are measuring same economic dimension, high correlation can interfere the results or the difference in ratios can be so small, that the selection between two or more ratios into the model can be more or less random.

To analyse the models we divided the group of 31 original ratios into three very general dimension, namely liquidity (L), solidity (S), and profitability (P) measures. It is quite obvious that some of the ratios are rather measuring some other elements like effectiveness than any of these three, but to make the analysis more simple we did this rough classification. The stepwise model used for discriminant analysis selects two liquidity measures, one profitability measure and one solidity measure one year prior to failure. Two years prior to failure it selects one liquidity measure, three solidity measures and two profitability measures and three years prior to failure the corresponding measures are two liquidity and one solidity measure.

The stepwise selection model for logit analysis selects two liquidity measures and one solidity measure one year prior to failure. Two years prior to failure it selects one liquidity measure, two profitability measures, and one solidity measure. Three years prior to failure the model includes only two liquidity measures.

The genetic algorithm selects three liquidity measures, two profitability measures and one solidity measure one year prior to failure. Two years prior to failure it selects four liquidity, two profitability and one solidity measure and three years prior to failure the corresponding measures are ten liquidity, two solidity and three profitability measures.

The dominant measure (or shared dominance with another measure) one year prior to failure is a measure on liquidity for all three models. The dominant measure two years prior to failure is a measure on solidity for discriminant analysis and profitability for logit analysis while the genetic algorithm has selected most liquidity measures. Three years prior to failure the dominant measure is a liquidity measure for all the selection methods.

The results indicated that in case of failure prediction liquidity seems to play an important role. Despite of the selected method, it was included in every model 1, 2, and 3 years prior to failure. Three of the nine models did not include any profitability measures and one excluded all solidity ratios. The explanation to this can be found by studying the concept and definition of failure in Finland.

The failure of a firm can in Finland be concequence of two different juridic process. First, if stockholders' equity in the balance sheet declines below 1/3 of the stock capital due to losses (see Finnish Companies Act), the firm goes into liquidation. This means that all the claims against the firm are settled. If the value of the company's debts is higher than the value of its assets, it is to be bankrupt. In an opposite case, it can continue its operations. Even though this process does not lead always to a failure of a liquidated firm, it is an obvious sign of continuing unprofitable operations. This type of failure is in this study called solidity bankruptcy.

Second, another failure type is if a firm cannot pay its debts when they fall due i.e. liquidity bankruptcy (see e.g. Finnish Bankruptcy Act). Both of these failure types are recognised in previous studies and they have both different theoretical frameworks, which form the basis to a financial ratio selection. Naturally, these failure processes are not mutually exlusive, i.e. decreased liquidity can be connected with solidity bankruptcy or signs for preconditions to solidity bankruptcy can be in context of liquidity bankruptcy.

From these two failure types the latter one occurs more often. Even though the preconditions for solidity bankruptcy exist, a firm can avoid failure process if it is not reporting that the stockholders' equity in the balance sheet has declined below 1/3 of the stock capital due to losses. Opposite to this, liquidity bankruptcy can not be avoided because the process is always started by creditors. This can be reflected in our models, which all included variables measuring liquidity.

The result can also be explained by the fact that in the original data group of liquidity ratios consisted of 19 ratios when solidity and profitability were presented only by six ratios each. Furthermore, the group of liquidity ratios is more or less mixed with alternative ratios measuring also other characteristics of a firm. In closer analysis this group should be divided into a smaller subgroups.

Factor analysis   To study further if the models really are measuring different economic characteristics of a firm we applied factor analysis using all variables included in original data one, two, and three years prior to failure, separately. This was done to find out if the variables in alternative models are describing different financial dimensions so that the selection of one variable into the model is not only a consequence of extremely small differences in the values of test statistics. Also, we were interested in more sophisticated classification for the original variables. The results of varimax rotated factor patterns for one year prior to failure are presented in Table 6. Note that factor analysis is based on linear relationships between the ratios just like linear discriminant analysis. However, it provides us with a method to identify alternative dimensions among the set of ratios.

Table 6. Factor loadings, rotated factor pattern with original variables one year prior to failure.

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6	FACTOR7
R1	0.14981	-0.08547	-0.01460	**0.86856**	0.12686	0.34859	-0.03770
R2	**0.78612**	-0.33028	-0.19238	0.03617	-0.00223	0.26984	-0.18610
R3	**0.94953**	-0.16159	-0.02586	0.07082	-0.04073	0.11173	0.13603
R4	**0.86741**	-0.35486	-0.09566	0.03230	0.06629	0.12234	-0.11180
R5	0.01967	-0.02379	0.28634	**0.92559**	0.09130	0.02386	0.05146
R6	0.07935	-0.07291	0.08122	**0.93324**	0.14829	-0.05255	0.03429
R7	0.23658	-0.32322	-0.01812	0.07722	-0.07020	**0.86354**	0.03647
R8	-0.05311	-0.06434	**0.92691**	0.26092	-0.00676	0.04368	0.05725
R9	-0.13186	-0.25326	**0.59222**	-0.09560	0.24413	-0.07906	0.31638
R10	-0.28072	**0.87795**	0.01347	-0.08977	-0.03123	-0.20723	-0.08578
R11	**0.58225**	-0.42641	0.10820	0.11140	-0.08536	0.06071	0.31040
R12	0.53782	**-0.57262**	0.02171	0.05981	0.07854	0.14483	0.45077
R13	0.00315	0.01701	**0.81010**	0.02442	-0.46193	-0.07141	0.03963
R14	-0.26590	**0.87481**	-0.00928	-0.08336	-0.03768	-0.15525	-0.11316
R15	0.48808	**-0.73847**	0.02116	0.06048	0.02561	0.20633	-0.01049
R16	-0.28664	**0.88468**	0.02008	-0.08609	-0.03430	-0.19925	-0.07250
R17	**0.92447**	-0.21929	0.02955	0.02326	0.04126	0.15045	0.13257
R18	0.07460	0.15021	-0.27203	0.11659	**0.86179**	0.05147	-0.03494
R19	0.04894	0.04462	**-0.73450**	-0.22312	0.25161	-0.07770	0.13485
R20	**0.94239**	-0.07172	0.00767	0.03535	-0.05068	0.13954	0.16748
R21	**0.59926**	-0.36552	0.01658	-0.03265	0.09482	-0.04187	-0.29366
R22	0.21395	-0.16042	-0.04928	0.22530	0.55841	**0.69537**	-0.02029
R23	-0.09100	-0.12504	**0.61812**	0.40235	0.50691	0.15204	0.04985
R24	-0.11212	-0.16625	0.06026	0.14841	**0.93992**	-0.00287	0.06995
R25	**0.60710**	-0.03399	-0.09079	0.00401	0.00203	0.11893	0.63130
R26	0.09686	**-0.85538**	0.15757	0.05207	0.00364	-0.05061	0.02538
R27	-0.37181	**0.60955**	-0.00168	0.12684	-0.04118	-0.29679	0.23711
R28	**-0.71866**	0.47460	-0.01226	-0.08076	0.05353	-0.09317	-0.24152
R29	0.32907	-0.30210	0.23939	0.16496	0.13410	**0.55929**	0.54319
R30	-0.15527	-0.20205	**-0.43476**	0.27436	-0.10342	-0.01340	0.09310
R31	0.48614	-0.35848	0.19786	0.07421	0.01557	**0.59937**	0.33863
	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6	FACTOR7
Proportion	0.3791	0.1376	0.1035	0.0777	0.0552	0.0443	0.0375
Cumulative	0.3791	0.5167	0.6202	0.6979	0.7531	0.7974	0.8349

The criterion based on Eigenvalues higher than one yielded a seven factor solution. The variables in all three models were loaded on four factors, i.e. on the first, second, third, and fourth. Variables in DA model as well as in GA model were representing all these four factors, but logit model included only two of them, namely the first and the fifth factors. The names of the factors are based on the ratios with highest loading on the factors. Factor one can be named as *Profitability and dynamic leverage factor*. This factor was represented in all models. The second factor, *Static leverage factor*, as well as the third factor, *Turnover factor*, were both included in DA and GA models but excluded in logit model. Finally, all models included variables loaded on factor five, *Quick assets factor*.

For the factor solution in one year prior to failure in the data could be identified also three other factors (the fourth, sixth, and seventh factors) which did not include any variables selected to the models. Two of them were named as *Cash factor* (factor 4) and *Static liquidity factor*, (factor 6). The seventh factor had only one high loading, on variable *Rate of return to common stock.*

The analysis may indicate that sepwise logit model uses less information than two other variable selection methods examined in this study. In the logit model there are a smaller number of variables and dimensions than in the other models.

One and three years prior to failure logit model uses variables only from two factors and two years prior to failure from three factors. Summarizing, the number of variables included into the models as well as the information content of the models are affected by the model selection method. Furthermore, connected with alternative prediction methods (DA, logit analysis, neural networks) they also lead to the different number of Type I errors, Type II errors and the total prediction accuracies as can be seen in the next section. Factor solutions for two and three years prior to failure are presented in Appendix 1.

6 Prediction results

In previous paragraphs we presented separate models for each year and each technique. It was noticed that the underlying assumption concerning the relationships between independent variables affects the model selection process in a prominent way. When the three alternative models seem all to use different information, the interesting question is if there are differences in their prediction ability. To study further the consequences of different model selection approaches we have applied corresponding statistical method to test the predictive ability of constructed models. In Table 7 the cross-validated prediction accuracy results are presented for every technique separately.

Table 7. Cross-validated prediction results for discriminant analysis (DA) and logit analysis (Logit), and neural networks (NN) prediction results.

	Type I error			Type II error			Total error		
year	DA	Logit	NN	DA	Logit	NN	DA	Logit	NN
1	13.51%	13.51%	**5.26%**	16.22%	13.51%	**0%**	14.86%	3.51%	**2.70%**
2	**24.32%**	27.03%	26.32%	**18.92%**	29.73%	27.78%	**21.62%**	28.40%	27.03%
3	16.22%	16.22%	**5.26%**	37.84%	35.14%	**27.78%**	27.03%	25.70%	**16.22%**

One year prior to failure the genetic algorithm based model used in neural networks perform better than the two other models. It produces only 5.26% type I errors and 0% type II errors while both logit analysis and discriminant analysis produces 13.51% type I errors, and 16.22% and 13.51% type II errors, respectively. The overall errors amount to only 2.70% for neural nets but to 14.86% and 13.51% for discriminant analysis and logit analysis.

Two years prior to failure the model with fewest errors was constructed using the stepwise selection method for discriminant analysis. For both error types it leads to the lowest misclassification error. The logit model lead to the highest misclassification rate.

Three years prior to failure the best classifier is again the genetic algorithm based model. The type I error is remarkably low 5.26%. The type II errors amount to 27.78%, the same amount as two years prior to failure, and the overall performance is also best with total errors amounting to 16.22% compared to 27.03% for discriminant analysis and 25.70% for logit analysis.

The results for the neural networks/genetic algorithm model are also better than the results of our earlier study (Back et al., 1994) where we received 10.51% type I errors, the type II error being equal in both studies, i.e. 0% one year prior to failure. In that study instead of using genetic algorithms we selected one financial ratio presenting liquidity, one solidity ratio, and one measure of profitability in our model.

8. Conclusions

The failure prediction research has suffered from the lack of any unified theory since the 1930's when first empirical studies on this subject were published. In spite of that, empirical prediction results have been promising. Without theoretical background alternative models have predicted the future of a firm usually correctly in 80% of the cases, in some studies the amount of correct classifications is even higher. The problem is that before the theoretical construction for failing firms is settled, the prediction accuracy is dependent on the best possible selection of variables included in to prediction models and also on the statistical method that is used.

Until 1980's the prominent method in failure prediction was discriminant analysis. In 1980's logistic analysis replaced this method and today even logistic analysis have some challengers. One of these are neural networks which seem to lead to higher prediction accuracy compared to the two other methods. In this study, we have compared these three central methods and also suggested a new possibility to be used in model selection, i.e., genetic algorithms. While stepwise ratio selection procedures have already been constructed for DA and logit, the empirical ratio selection for neural networks has been an open question.

This study shows that the use of DA, logit analysis or genetic algorithm all lead to different failure prediction models. The amount of variables included in the models varies. Also, different methods lead to the selection of different financial ratios. Despite of the selection method used, liquidity seems to be very important factor in failure prediction. Two reasons for this were discussed. First, the liquidity failure is the more general failure type in Finland which stress the importance of this factor in the models. Second, the variables in our original sample were mostly factors describing liquidity.

In this study the group of original variables was formed by selecting those variables which in previous central studies have been found good predictors of failure. These variables were then roughly divided into three categories, namely profitability, solidity, and liquidity. To analyse further the constructed models factor analysis was done. It indicated that in addition to the different number of variables in different models also the information content of the models varied. In all three years prior to failure the stepwise model selection for the logit model used the information connected to the fewest number of factors. The number of factors in factor solutions, 7-8 factors each year indicated also that the group of original ratios must be divided into more than three categories.

Furthermore, the prediction accuracy of selected models was tested using corresponding statistical methods for DA and logit analysis and neural networks for genetic algorithms. The results indicated that neural networks outperformed two other methods one and three years prior to failure. The misclassification rate one year prior to failure was extremely low, only 2.7%. Two years prior to failure traditional discriminant analysis led to a lowest misclassification rate.

In summary, three conclusions can be made. First, the differences between alternative model selection methods affect the number of independent variables to be selected. Second, not only the number of variables but also the information content of the models varies due to the variables that are measuring different economic dimensions of a firm. Finally, connected with alternative failure prediction methods, also the prediction accuracy varies.

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APPENDIX 1

A. Factor analysis with original variables two years prior to failure.

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6	FACTOR7
R1	-0.12221	0.18164	0.35278	-0.10284	0.85057	-0.06228	-0.11239
R2	-0.21501	0.70113	0.56922	0.07832	0.11152	0.07008	-0.20569
R3	-0.29971	0.86710	0.21874	0.01211	0.07582	0.11827	-0.04553
R4	-0.24195	0.86335	0.30905	0.01403	0.09791	0.10415	-0.11318
R5	-0.12645	0.22308	0.09925	0.35476	0.78230	0.17623	-0.02431
R6	-0.14801	0.20982	0.03313	-0.08883	0.89376	0.16198	0.12060
R7	-0.16859	0.10473	0.84759	0.00686	0.32708	-0.12879	0.01045
R8	0.00001	-0.00492	0.16788	0.95092	-0.01379	0.04649	0.14991
R9	-0.09809	-0.01048	0.12571	0.15928	-0.01142	0.13233	0.86330
R10	0.91381	-0.22729	-0.11416	-0.03934	-0.08468	-0.08265	-0.10050
R11	-0.33952	0.20975	0.69753	0.21293	-0.09947	0.04867	0.19992
R12	-0.60284	0.29688	0.62457	0.17274	0.05112	0.09649	-0.05540
R13	0.05251	-0.08019	-0.03207	0.83162	-0.06615	-0.35392	0.21344
R14	0.90007	-0.22552	-0.10193	-0.05267	-0.03927	-0.06689	-0.19626
R15	-0.61568	0.52887	0.45527	0.05532	0.13653	-0.06150	-0.10093
R16	0.90143	-0.22951	-0.11125	-0.04748	-0.11227	-0.10124	-0.04228
R17	-0.29504	0.81329	-0.12670	-0.01676	0.19227	0.05549	0.10195
R18	0.01539	0.15698	-0.00320	-0.19418	0.12260	0.87539	-0.13161
R19	0.21081	0.03147	-0.06794	-0.77018	-0.04020	0.22536	0.08801
R20	-0.11435	0.81765	0.32704	-0.02584	0.07765	0.14051	0.06516
R21	-0.12493	0.64090	0.04448	-0.05427	0.04954	0.08439	0.12355
R22	-0.12076	0.24417	0.75745	0.04559	0.25153	0.50106	-0.04418
R23	-0.06120	0.08454	0.34234	0.75789	0.05206	0.49704	0.02351
R24	-0.07208	0.19012	0.11328	-0.06338	0.12473	0.88376	0.25424
R25	-0.36286	0.45085	-0.08202	0.11721	0.31745	0.22207	0.19534
R26	-0.80277	0.28643	0.20070	0.05090	0.11082	-0.15075	-0.03643
R27	-0.50748	-0.42863	0.17835	0.06737	-0.15432	0.06566	0.15462
R28	0.71686	-0.37631	-0.35031	-0.03911	-0.11216	0.01654	0.16333
R29	-0.26915	0.09559	0.83345	0.24411	0.07704	0.10040	0.20736
R30	-0.07822	0.13624	-0.07974	0.14554	-0.36461	-0.14570	0.35766
R31	0.50476	-0.01189	-0.29066	0.05665	-0.15314	0.06181	-0.42615

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6	FACTOR7
Proportion	0.3667	0.1225	0.0922	0.0702	0.0651	0.0601	0.0373
Cumulative	0.3667	0.4892	0.5814	0.6516	0.7167	0.7768	0.8141

B. Factor analysis with original variables three years prior to failure.

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6	FACTOR7	FACTOR8
R1	-0.09289	0.12733	-0.07481	0.88475	0.32589	0.09594	-0.14962	-0.04109
R2	-0.26891	0.71077	-0.09207	0.09224	0.17482	-0.10763	-0.42017	-0.00447
R3	-0.25975	0.89706	-0.07605	0.16601	-0.01744	0.07267	-0.10492	-0.10097
R4	-0.32167	0.88423	-0.06275	0.14127	0.03504	0.02488	-0.21970	-0.04366
R5	-0.02109	0.06198	0.35881	0.87845	-0.00263	0.14042	-0.02846	-0.07559
R6	-0.01004	0.12386	-0.04410	0.94486	-0.04066	0.16770	0.04187	0.02926
R7	-0.27074	0.05752	-0.01157	0.05303	0.89167	-0.12183	-0.06737	0.03347
R8	-0.01138	-0.13967	0.96236	0.00581	0.01370	-0.02120	0.10040	0.05811
R9	-0.08349	-0.03103	0.19720	-0.10107	0.01999	0.10364	0.78591	0.24033
R10	0.87167	-0.15118	0.05799	-0.09856	-0.10157	-0.03804	-0.29738	0.03787
R11	-0.63181	0.12279	0.15746	-0.13793	0.24896	-0.00775	0.19613	0.11235
R12	-0.88313	0.07778	0.12962	0.04495	0.16890	0.09053	-0.20984	-0.07360
R13	0.02487	-0.21713	0.79017	-0.15474	-0.03321	-0.37066	0.08872	0.10562
R14	0.85505	-0.13454	0.06426	-0.08626	-0.06795	-0.03184	-0.34666	0.04269
R15	-0.83922	0.24048	0.05252	0.01564	0.22652	-0.08212	-0.27048	0.04346
R16	0.87851	-0.15978	0.04876	-0.09955	-0.10954	-0.04381	-0.27904	0.03082
R17	-0.21313	0.91107	-0.04810	0.10211	0.02413	0.09542	0.12577	-0.08256
R18	0.07814	0.02350	-0.25052	0.15401	-0.03242	0.85748	-0.14224	-0.03011
R19	0.15753	0.07248	-0.73769	-0.16037	-0.10885	0.34801	0.04877	0.12534
R20	-0.01293	0.88007	-0.14486	0.06529	0.11604	-0.00305	0.11768	-0.10271
R21	-0.14274	0.55962	-0.02539	-0.05475	-0.09549	0.11890	0.24506	-0.00808
R22	-0.11214	0.13369	0.01126	0.27459	0.71306	0.56319	-0.01150	-0.01006
R23	-0.05291	0.00933	0.82600	0.20615	0.06782	0.42728	0.07825	-0.02315
R24	-0.00710	0.12855	0.02192	0.18835	0.00845	0.86880	0.29668	0.13743
R25	-0.40386	0.28969	-0.16198	0.19850	-0.07067	0.11102	0.01987	-0.58243
R26	-0.90555	0.06427	0.12506	-0.00409	0.07288	-0.10375	-0.04710	0.02628
R27	-0.14213	-0.56359	0.01690	0.06363	-0.01231	-0.00896	0.02361	-0.39774
R28	0.88724	-0.17279	0.04478	0.00895	-0.09449	0.06063	0.23558	0.01252
R29	-0.43810	-0.08217	0.19157	0.04231	0.80305	0.03619	0.11375	-0.05686
R30	-0.19155	-0.02513	-0.05792	0.04908	-0.07199	0.11914	0.18797	0.79386
R31	0.40888	-0.17820	0.04837	-0.01472	-0.47566	0.20589	-0.48931	0.28643

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6	FACTOR7	FACTOR8
Proportion	0.2874	0.1493	0.1116	0.0819	0.0720	0.0553	0.0423	0.0363
Cumulative	0.2874	0.4367	0.5483	0.6301	0.7021	0.7574	0.7997	0.8361